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Mathematical Modeling of Operating Process Parameters of a Mixed-flow Turbofan Engine with an Afterburner

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ABSTRACT

The article provides a brief analytical review of mathematical models of the working process used to study the parameters and characteristics of a gas turbine engine at all stages of its creation and operation. It is shown that the first-level mathematical model can be used for the estimation of parameters and characteristics under real operating conditions that differ significantly from the calculated ones. A mathematical model of a mixed-flow turbofan engine with an afterburner is proposed in the article; the construction principle and structure of the mathematical model of the engine are given. The model allows solving various problems associated with calculating the *performance* of a mixed-flow turbofan engine with an afterburner in a wide range of external conditions and engine operating modes.

Keywords: Mathematical model; Turbofan engine; Afterburner.

INTRODUCTION

During the operation of aviation gas turbine engines (GTEs), their parts and assemblies are exposed to a wide range of loads and other destructive factors. This impact leads to a deterioration in the technical condition of the engine. Specific fuel consumption increases, the gas temperature before the turbine rises, gaps in the flow path increase due to wear and weathering of seals, the condition of the surface layer of the blades deteriorates, etc.

The need to ensure flight safety requires the development of a system for early detection of developing faults to exclude engine failures in flight. Such a system is a modern engine diagnostics system that allows the detection of faults in advance, which makes it possible to predict, with a certain probability, the time of reaching the limit state.

Objective determination of the operational status of a GTE and monitoring its changes during long-term operation require the measurement, recording, and subsequent processing of a large number of parameters that characterize the operation of the engine's assemblies and systems (Tahan *et al*. 2017).

Modern approaches to estimating the operational status of GTEs by parametric methods are based on the mathematical description of their working processes, encompassing both the engine as a whole and its individual units (Ahmed and Osipov 2020; Perevoschikov 2019; Raherinjatovo and Gishvarov 2017). Mathematical models of the engine are classified by the level of

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complexity, which is understood as the degree of detailed description of the working processes occurring in the engine and its units. There are five levels of complexity, which are numbered from zero to four (Titov and Osipov 2016).

In zero-level models, the GTE is treated as a "black box" where flight conditions serve as input parameters and the engine's functional characteristics as output parameters. These zero-complexity models are typically used when the GTE is considered as a component of the aircraft system. First-level complexity models use thermal gas dynamics equations with assumptions for one-dimensional flow, as well as additional relationships specific to certain engine designs. In contrast, higher-level complexity models utilize equations that describe three-dimensional flow, accounting for both laminar and turbulent boundary layers within the engine gas flow duct. These models are used during the design and development stages of the engine. When creating a mathematical model of an engine as a whole and determining the model's level of complexity, the defining factor is the complexity of the units that make up the engine, as well as the complexity of the mathematical models describing its individual units.

In most cases, conventional mathematical models of GTE operation, including zero and first-level complexity models, are used to estimate the parameters and characteristics of a GTE in operation (Derbel and Beneda 2019; Kamboukos and Mathioudakis 2005; Khoreva and Ezrokhi 2017).

Thermal and gas-dynamic engine parameters (pressure, temperature, etc.) in different sections of the engine gas duct, as well as engine output parameters (thrust, fuel consumption, rotor speed, etc.), provide essential information about the condition of the engine gas duct (Kofman 2007). In this case, the task of engine diagnostics can be solved within the framework of two approaches. According to the first approach, the decision about the state is made by comparing the measured parameters with the norm of their permissible deviations. If the parameter values are within the technical norms, the engine condition is classified as serviceable. If the parameter values are outside the established norms, the engine condition is considered as faulty. By monitoring the change of parameters during the engine operation, it is possible to anticipate the moment when the parameter goes beyond the established tolerance and to timely prevent engine failure in flight. The nature of changes in thermal and gas-dynamic parameters over the operating time allows determining the cause of the deterioration of the engine's condition. The second approach focuses on pinpointing defect locations and identifying internal causes of faults. This method employs mathematical models that describe the gas-dynamic processes within the engine.

During operation, when diagnosing a GTE by thermal and gas-dynamic parameters, the level of fault localization to the removable module is sufficient. This limitation allows the reduction of the number of independent parameters and simplifies the problem-solving process. A deeper level of fault localization is only advisable during the debugging and manufacturing phases of the engine. The practical implementation of the two approaches described above is carried out sequentially. First, the diagnostic problem at the first level is solved by analyzing changes in thermal and gas-dynamic parameters corrected for standard atmospheric conditions and a specific operating mode. If signs of parameter deviations from the basic values are detected, the second level is carried out to determine the location of the fault.

The diagnostic method using thermal and gas-dynamic parameters allows the detection of faults such as erosive wear of elements of the flow part, burnouts and warping of parts of the hot part of the engine, mechanical damage and destruction of parts, etc., at an early stage of development (Li 2009).

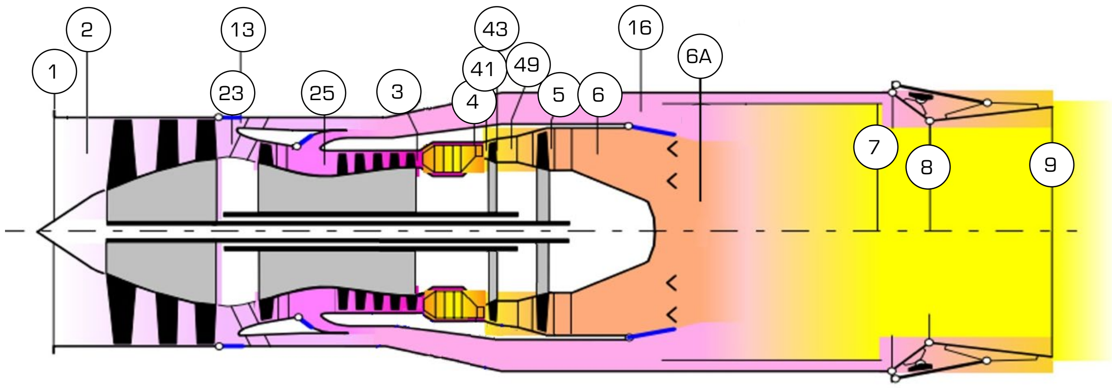
Purpose and objectives of research

To date, a general theory and approaches to solving the problems of diagnosing the technical condition of engines have been developed. At the same time, insufficient attention is paid to models and methods for analyzing the operating variables and technical condition at the stage of engine operation under conditions of uncertainty. Thus, the problem of the development of existing and the creation of new approaches and computational methods for solving nonlinear diagnostic problems under conditions of input data uncertainty remains relevant.

The purpose of the study was to develop a mathematical model of non-stationary operating modes of a mixed-flow turbofan engine with an afterburner. The model allows determining the dynamic characteristics of the engine, taking into account a limited number of parameters and requirements for the time and quality of transient processes, thereby increasing the efficiency of maintenance during operation and at the stage of engine debugging.

Object for mathematical description

The most common type of the "*turbofan engine*" with an "*afterburner*" is the mixed-flow engine, the diagram of which is shown in Fig. 1. The stations of the mixed-flow turbofan engine with an afterburner are labeled in accordance with Society of Automotive Engineers (SAE) notation.



Source: Elaborated by the authors.

Figure 1. Station numbering for a mixed-flow turbofan engine with an afterburner chamber. 1 = inlet to the air intake; 2 = exit from the air intake and entrance to the engine; 3 = exit from the high-pressure compressor (HPC) and entrance to the combustion chamber (CC); 4 = exit from the CC and entrance to the nozzle diaphragm of the high-pressure turbine (HPT) stage; 5 = exit from the axial low-pressure turbine (LPT) wheel; 6 = exit from the main flow nozzle of the turbofan engine; 7 = exit from the afterburner chamber and entrance to the jet nozzle; 8 = critical section of the jet nozzle; 9 = exit (cut) of the jet nozzle; 13 = output from the secondary flow of the turbofan engine; 16 = exit from the secondary flow nozzle of the turbofan engine; 23 = exit from the primary flow fan of the turbofan engine; 25 = entrance to the HPC; 41 = exit from the nozzle diaphragm of the HPT stage and entrance to the axial HPT wheel; 43 = exit from the high-pressure axial turbine wheel and entrance to the nozzle diaphragm of the LPT stage; 49 = exit from the nozzle diaphragm of the low-pressure turbine stage and entrance to the low-pressure axial turbine wheel.

Features of modeling of a mixed-flow turbofan engine with an afterburner

A peculiarity of modern turbofan engines is that the afterburner chamber is located behind the mixing chamber of the turbofan engine, in which the mixing of gases of the primary and secondary streams occurs. In such engines in most modes of operation, a small difference in gas pressure in the secondary stream and in the mixing chamber is maintained. This leads to the fact that the processes occurring in the afterburner chamber have a significant effect on the fan operation mode. The mixing of cold air from the secondary flow with hot gas coming out of the turbine leads to the process of fuel ignition in the afterburner chamber at low temperatures, which worsens the combustion conditions. The operation of the afterburner chamber significantly affects the fan operation mode. To ensure a given fan operation mode, it is advisable to control the critical cross-section of the jet nozzle by the parameters that most fully reflect the fan operation mode. Such parameters include gas dynamic functions in the secondary flow behind the fan.

The creation of a mathematical model will make it possible to study the influence of the characteristics of individual components on the characteristics of the engine as a whole. The use of mathematical modeling makes it possible to calculate the altitude-velocity and throttle performances of the engine throughout the entire range of aircraft flight modes under both standard atmospheric conditions and their deviation from standard conditions, to trace the remaining life of the engine during operation and to determine the most optimal ways to compensate for changes in the parameters of the units and the engine as a whole.

Before creating a model, a parametric study should be carried out to identify the physical nature of the relationships between engine parameters and to evaluate their influence on thrust, economic efficiency and specific mass characteristics (Ulitenko 2019). The effect of engine bypass ratio and turbine inlet temperature on the throttle performances of a turbofan engine is ambiguous. As the bypass ratio increases, the specific thrust of the engine decreases and, consequently, the airflow required to obtain a given thrust increases.

On the other hand, as the bypass ratio increases, the specific thrust decreases, which, in turn, leads to a decrease in absolute thrust. A similar trend is observed when analyzing the effect of the turbine inlet temperature on the throttle performances of a turbofan engine with an afterburner. In studies, it is shown that with decreasing efficiency of the units and increasing losses in the flow duct, the optimum values of the cycle parameters (gas temperature upstream of the turbine, total pressure ratio, and bypass ratio) decrease (Grigoriev *et al.* 2009; Popov 2007). In the study by Kuzmichev *et al.* (2016), generalized results of calculations on the influence of the efficiency level and loss coefficients on the optimum values of the total pressure ratio and the bypass ratio of a turbofan engine are presented. In this case, the relative change of the considered parameters is understood as the ratio of optimal values for the given initial data. It is shown that the reduction in these parameters depends significantly on the value of the gas temperature upstream of the turbine.

During flight, the functional parameters of the turbofan engine with mixed exhaust and afterburner are repeatedly changed. Every few seconds, the airplane evolves, its speed (Mach number) and flight altitude change. As a result, rotor speed, air and fuel flow rates, air temperature behind the compressor, and gas temperature in front of the turbine and at the afterburner exit change. On average, 5-8 transient unsteady operating modes are observed in 1 minute. This means that for most of the flight time, this type of engine operates in transient modes. Therefore, the development of mathematical models describing transient processes in GTE is highly relevant (Rath *et al*. 2023).

Mathematical model

The mathematical model of the engine links the dependent parameters with the independent parameters of the units. Thus, the task of diagnostics is to determine the deviations of independent parameters from their base values based on the results of monitoring the dependent parameters, only a part of which can be measured in operation. In general, the mathematical model of an engine can be represented by a vector Eq. 1:

$$\Phi(\mathrm{YB})=0 \tag{1}$$

where Y is the vector of dependent variables and B is the vector of independent parameters.

The components of the Y-vector are corrected rotational speeds of the rotors of the low-pressure compressor (LPC) and HPC, airflow rate through the compressors, total pressure ratio across the turbine stages, and total temperature at the outlet of the CC. The components of the B-vector are mass flow rate of the working fluid through the turbine and HPC, power at the rotor shafts, cross-sectional area of the jet nozzle, and total pressure at the inlet to the mixing chamber.

Linearization of the mathematical model equations leads to a system of algebraic equations in deviations (Eq. 2):

$$\delta y = MAT\delta b_j\,, \tag{2}$$

where δy is the vector of deviations from the basic value of the controlled parameter y_{base}, $\delta y_i = \frac{y_i - y_{basei}}{y_{basei}}$, $i = 1 \div k$, k is the number of controlled parameters, δb_j is the vector of parameter deviations, $j = 1 \div n$, n is the number of independent parameters, and MAT is the matrix of influence coefficients. If the number of controlled parameters exceeds the number of independent parameters, the problem is reduced to solving the initial system of equations using the least squares method (Li and Korakianitis 2011).

The peculiarities of modeling transient modes of a mixed-flow turbofan engine with an afterburner are discussed in Panin *et al.* (2000) and Williams and Ezunkpe (2024). One of these peculiarities is the change in the pressure ratio at the LPT when flight conditions or jet nozzle geometry change (Golubev 1993).

Assuming that the mass flow parameter (MFP) in the nozzle apparatus of the first stage of a LPT is independent of flight conditions, the following Eq. 3 is valid:

$$PR_{LPT} = \sqrt{\frac{T_5}{T_{43}}} = \frac{A_6 MFP(M_6)}{A_{49} MFP(M_{49})} \tag{3}$$

where PR_{LPT} is the ratio of pressure across a LPT, T_5 is the total temperature at the output of the turbine, T_{43} is the total temperature at the inlet of the LPT, A_{49} is the area of the inlet mixer in the core stream, A_6 is the area of the annular cross-section of the nozzles of the first stage LPT, and MFP is calculated using Eq. 4

$$MFP(M) = \frac{M\sqrt{\gamma / R}}{\left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}}, \tag{4}$$

where M is the Mach number, R is the gas constant, and γ is the ratio of specific heats.

In a mixed-flow turbofan engine, one important condition is the equality of static pressures at the inlet of the mixing chamber (Muhamedov 2014). When this condition is satisfied, and applying the pressure balance equation for both the core flow and bypass flow, we derive the following dependence (Eq. 5):

$$(\pi_r)_{16} = \frac{\eta_{rb} PR_{LPC} \eta_{rd} PR_{HPC}}{\eta_{rF} PR_{fan} PR_{HPT} PR_{LPT}} (\pi_r)_6, \tag{5}$$

where $(\pi_r)_6$ and $(\pi_r)_{16}$ are the compressible flow functions (ratio of pressures at sections 6 and 16), η_{rb}, η_{rF}, and η_{rd} are the *total* pressure recovery coefficients in the CC, in the fan flow, and in the bypass flow, PR_{LPC}, PR_{HPC}, and PR_{fan} are the ratio of pressures across the LPC and HPC, the fan in the secondary flow, PR_{HPT} and PR_{LPT} are the ratio of pressures across the HPT and LPT.

If the pressure ratio across the bypass fan flow is equal to the pressure ratio across the LPC, which is typical for modern engines with low bypass ratios, then Eq. 5 simplifies to the following (Eq. 6):

$$(\pi_r)_{16} = \frac{\eta_{rb}\eta_{rF} PR^*_{HPC}}{\eta_{rd} PR^*_{HPT} PR^*_{LPT}} (\pi_r)_6 \tag{6}$$

The matching of the powers developed by the HPT and consumed by the HPC in nonstationary regimes is described by the equation of motion. The extended form of this equation is presented in Panin *et al.* (2005) and is expressed as follows (Eqs. 7 and 8):

$$\frac{PR_{HPC}}{MFP(M_{25})} = C_1 \sqrt{\frac{(1+f)}{\eta_{HPT}\eta_m}\left[\frac{PR_{HPC}^{\frac{\gamma-1}{\gamma}}-1}{\eta_{HPC}} + I_{PI}\frac{N_{HP}dN_{HP}}{d\tau}\frac{1}{C_2 p_{t25}\sqrt{T_{t25}}MFP(M_{25})}\right]} \tag{7}$$

where the coefficients are:

$$C_1 = \frac{A_{25}\sqrt{C_p\Big|_{T_{25}}^{T_3}}}{MFP\left(M_{41}\right)A_{41}\eta_{rb}\sqrt{c_p\left[1-\frac{1}{\left(PR_{HPT}\right)^{\frac{\gamma_g-1}{\gamma_g}}}\right]}}, \quad C_2 = \Big|_{T_{25}}^{T_3} A_{25}\frac{900}{\pi^2} \tag{8}$$

where $C_p\Big|_{T_{25}}^{T_3}$ is the average specific heat in the temperature range $T_{25}-T_3$, A_{25} is the area of inlet to the HPC, A_{41} is the area of exit from the HPT stage nozzle and inlet to the HPT axial wheel, τ is the time, γ_g and γ are the ratios of specific heats of gas and air, C_p is the specific heat during the combustion process (the model employs temperature-dependent properties of gas for more accurate calculations), η_{HPT} is the efficiency of the HPT, η_{HPC} is the efficiency of the HPC, η_m is the mechanical efficiency of the turbocharger coupling, I_{PI} is the polar moment of inertia of the core spool, N_{HP} is the rotational speed of the high-pressure rotor, and p_{t25} and T_{t25} are the total pressure and temperature at section 25.

Taking into account $\dot{m} = (1+\alpha)\dot{m}_c$, the power balance condition of the LPC and LPT in unsteady mode is expressed as Eqs. 9, 10:

$$(1+f)c_{p49}T_{t49}l_{LPT}\eta_m-(1+\alpha)c_{p25}T_{t25}l_{LPC}=I_{LP}N_{LP}\frac{dN_{LP}}{d\tau}\frac{\sqrt{T_{t25}}}{MFP(M_{25})p_{t25}A_{25}}\,, \tag{9}$$

$$l_{LPT}=\left(1-\frac{1}{PR_{LPT}^{\frac{\gamma_{49}-1}{\gamma_{49}}}}\right)\eta_{LPT}\,,\; l_{LPC}=\left(PR_{LPC}^{\frac{\gamma_{25}-1}{\gamma_{25}}}-1\right)/\,\eta_{LPC} \tag{10}$$

where I_{LP} is the polar moment of inertia of the low-pressure spool and N_{LP} is the rotational speed of the low-pressure rotor.

From the equation of conservation of energy, the total temperature of the mixer T_{t6A} is given Eq. 11: (Tereshchenko *et al*. 2004):

$$T_{t6A}=T_{t6}\left[\frac{c_{p6}}{c_{p6A}}\frac{1+\alpha'\left(c_{p16}/c_{p6}\right)\left(T_{t16}/T_{t6}\right)}{1+\alpha'}\right], \tag{11}$$

where α' is the mixer bypass ratio and c_{p6A} is the specific heat of the mixture, $c_{p6A}=\dfrac{\alpha' c_{p16}+c_{p6}}{1+\alpha'}$.

The engine bypass coefficient α is related to the mixer bypass coefficient α' by the relation $\alpha'=\dfrac{\alpha}{1+f}$.

Based on the principle of conservation of mass and the definition of α', we can write for an inflated exhaust nozzle and inlet nozzles of an LPT as Eq 12:

$$1+\alpha'=\frac{p_8A_8}{\sqrt{T_8}}\frac{\sqrt{T_{49}}}{p_{49}A_{49}}\frac{\Gamma_8}{\sqrt{R_8}}\frac{\sqrt{R_{49}}}{\Gamma_{49}}\,, \tag{12}$$

where $\Gamma=\sqrt{\gamma\left(\dfrac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$.

The impulse function can be defined as Eqs. 13–15:

$$I=pA\left(1+\gamma M^2\right) \tag{13}$$

or

$$I=\dot m\sqrt{\gamma\,\Phi(M,\gamma)}\,, \tag{14}$$

where

$$\Phi(M,\gamma)=M^2\left\{\left[1+(\gamma-1)/2\right]M^2\right\}. \tag{15}$$

The solution of Eq. 13 for $\Phi(M_{6A},\gamma_{6A})$ gives Eq. 16:

$$\Phi(M_{6A},\gamma_{6A})=\left[\frac{1+\alpha'}{\dfrac{1}{\sqrt{\Phi(M_6,\gamma_6)}}+\alpha'\sqrt{\dfrac{R_{16}\gamma_6}{R_6\gamma_{16}}\dfrac{T_{t16}/T_{t6}}{\Phi(M_{16},\gamma_{16})}}}\right]^2\frac{R_{6A}\gamma_6}{R_6\gamma_{6A}}\tau_m\,, \tag{16}$$

where

$$\tau_m=\frac{T_{6A}}{T_6}=\frac{c_{p6}}{c_{p6A}}\frac{1+\alpha'(c_{p16}/c_{p6})(T_{16}/T_6)}{1+\alpha'} \tag{17}$$

For the given value M_6 and M_{16}, Eqs. 16 and 14 yield a quadratic equation with the following solution (Eq. 18):

$$M_{6A}=\sqrt{\frac{2\Phi}{1-2\gamma_{6A}\Phi+\sqrt{1-2(\gamma_{6A}+1)\Phi}}}\,. \tag{18}$$

The mixer total pressure, in terms of τ_m and other flow properties at stations 6, 16, and 6A is obtained directly from the MFP (Eq. 19):

$$p_{t6A}=p_{t6}\frac{(1+\alpha')\sqrt{\tau_m}\,MFP(M_6,\gamma_6,R_6)}{1+A_{16}/A_6 MFP(M_{6A},\gamma_{6A},R_{6A})}\,. \tag{19}$$

Writing the mass flow equation for stations 2 and 9 in terms of flow properties and MFP gives (Eq. 20):

$$\left(\frac{p_{t9}}{\sqrt{T_{t7}}}\right)A_9 MFP\left(M_9\right)=\dot{m}_2\left(1+f_b+f_{ab}\right), \tag{20}$$

where

$$M_9=\sqrt{\frac{2}{\gamma_{ab}-1}\left[\left(\frac{p_{t9}}{p_9}\right)^{\frac{\gamma_{ab}-1}{\gamma_{ab}}}-1\right]}\,. \tag{21}$$

The sum $\left(f_b+f_{ab}\right)$ can be determined from the energy balance (Eq. 22):

$$\eta_{b\Sigma}\left(\dot{m}_{fb}+\dot{m}_{fab}\right)h_{pR}=\dot{m}_2\left(f_b+f_{ab}\right) \tag{22}$$

where $\eta_{b\Sigma}$ is the total efficiency of fuel combustion in the CC and afterburner chamber, and $\dot{m}_{fb}$ and $\dot{m}_{fab}$ are the mass flow rates in the main CC and afterburner, $\dot{m}_2$ is the mass flow rate at the engine inlet, and h_{pR} is the lower calorific value of fuel.

Proceeding from Eq. (23)

$$f_b+f_{ab}=\frac{\eta_{b\Sigma}\left(\dot{m}_{fb}+\dot{m}_{fab}\right)h_{pR}}{\dot{m}_2}\,. \tag{23}$$

The static pressure in the outlet section of the jet nozzle is (Eq. 24):

$$p_9=p_{t6A}\left(\pi_r\right)_9\pi_{ab}\,. \tag{24}$$

The value π_{ab} is determined from Eq. 25:

$$\pi_{ab}=\eta_{vl}\eta_{heat}\,, \tag{25}$$

where η_{vl} is the viscous loss coefficient and η_{heat} is the total pressure loss coefficient during combustion. The hydraulic coefficient η_{vl} is determined from the experimental characteristics of the afterburner chamber depending on the Mach number in the mixing chamber (Eq. 26):

$$\eta_{vl}=f\left(M_{6A}\right). \tag{26}$$

The magnitude η_{heat} is defined as a function (Khoreva and Ezrohi 2017) as shown by Eq. 27:

$$\eta_{heat}=f\left(\frac{T_{t7}}{T_{t6A}},\ M_{6A}\right) \tag{27}$$

The exhaust nozzle impulse can be obtained from the Eq. 28:

$$I = A_9\left(\frac{p_9}{r(M_9)} - p_{amb}\right) \tag{28}$$

where p_{amb} is the static pressure of atmospheric air and r(M9) can be obtained by Eq. 29:

$$r(M_9) = \frac{1 - \dfrac{((\gamma_9 - 1)/2)M_9^2}{\left[1 + ((\gamma_9 - 1)/2)M_9^2\right]}}{1 + \dfrac{((\gamma_9 + 1)/2)M_9^2}{1 + ((\gamma_9 - 1)/2)M_9^2}} \tag{29}$$

Engine thrust F (Eq. 30) corresponding to the real jet nozzle exit (Kulyk *et al.* 2018):

$$F = I - \dot{m}_2 V_0 \tag{30}$$

where V_0 – is the flight speed.

The total fuel-air ratio can be expressed as the ratio of CC and afterburner fuel-air ratios as (Eq. 31):

$$f_0 = \frac{f_b}{1+\alpha} + f_{ab} \tag{31}$$

The specific fuel consumption per thrust S is (Eq. 32):

$$S = \frac{f_0}{F/\dot{m}_1}. \tag{32}$$

The values of the parameters of individual units obtained in accordance with the presented method are used to analyze the causes of engine thrust loss during its operation and to develop methods for compensating for these losses, as well as to determine the condition of its units.

Control laws (Eq. 33):

$$\begin{cases} T_{t4} - T_{0t4} = 0 \\ T_{t7} - T_{0t7} = 0 \\ A_8 - A_{08} = 0 \end{cases}. \tag{33}$$

The proposed method allows determining the parameters of a turbofan engine only based on those parameters that are measured in the electronic automatic control systems of the turbofan engine and the aircraft, without installing additional receivers and sensors.

DISCUSSION

In the operation of aircraft systems, diagnosing the engine operating condition is one of the most challenging tasks. Currently, there is no universally accepted mathematical model for a turbofan engine as a basic one. An analysis of the capabilities of modern mathematical models by the range of solved tasks, forms of initial data representation, functionality, complexity, and other key criteria has shown the necessity of developing a mathematical model that includes self-learning functions.

The level of mathematical models largely determines the range of problems that can be solved using them. The significant complexity of the diagnostic object – aircraft engine – imposes special requirements on the technologies for creating the means of engine support in operation. This concerns both software and engineering means. It is always necessary to make a prognostic assessment: whether the development team is ready financially and, most importantly, skillfully to continue working in this direction, especially if the benefits are minimal and the costs exceed expectations.

The use of digital interfaces and data acquisition, storage, and processing systems has created the conditions for combining disparate control technologies into an integrated information environment. Such integrated solutions have already been developed and are actively implemented by aircraft manufacturers. These systems are built according to a single principle.

During the processing of the obtained data, indicators characterizing the current and predicted condition of the engine are calculated. Expert assessments are made based on all necessary information and data from similar problem cases stored in knowledge bases. Systems with centralized data offer numerous advantages for both aircraft manufacturers and aircraft operators. Manufacturers gain access to crucial information for optimizing the maintenance and logistics of engines, enhancing reliability and service life, improving design, and introducing new models. In exchange for the data provided, the customers receive timely and accurate information on the engine status, which helps increase availability and reduce maintenance costs.

This approach to obtaining diagnostic information is quite complex in terms of the material resources involved and the competence of the researcher, which limits the possibility of its implementation. Therefore, the development of simplified diagnostic technologies with a high degree of readiness is so important.

The scheme of engines with a low bypass ratio, mixing chamber, and afterburner suggests their wide application in aviation (supersonic airplanes, unmanned aircraft systems, cruise missiles). Therefore, the proposed approach can be useful in both the design and creation of such systems and in the process of their operation. Besides, the expansion of the area of application of the mathematical model should not necessarily lead to its complication.

The proposed model can be used for predictive analytics and determining the future state of the object under various operating conditions, determining the pre-failure state of the system, and identifying unacceptable operation of the system in case of a delayed failure situation.

Additionally, the proposed approach involves the use of calculated parameters, which is an undeniable advantage for engines of low testability, as it can be implemented directly in the onboard engine system, bypassing the complex stage of ground-based processing and analysis of flight information.

The proposed mathematical model makes it possible to organize diagnostics by parameters inaccessible for direct measurement. On the one hand, the increase in the number of parameters obtained by direct measurement increases the informativeness of the mathematical model of the engine, and on the other hand, it requires an increase in the volume of diagnostic information stored in flight and its preliminary processing in order to increase the efficiency of fault localization and maintenance efficiency.

The use of mathematical models makes it possible to fundamentally change diagnostics and engine control methodology by transitioning to control by parameters that directly determine its main characteristics, but are not available for measurement (engine thrust and specific fuel consumption, gas temperature before the turbine, reserves of compressor gas dynamic stability).

CONCLUSION

- Nowadays, control technologies based on complex solutions and including means of measuring various diagnostic parameters and special software for centralized data collection and processing are being intensively developed. This allows ensuring high reliability of assessment and forecasting of technical conditions and, as a result, increases the efficiency of maintenance, reliability, and engine life.
- Construction of a mathematical model for diagnostics and assessment of a turbofan engine operational status has its own peculiarities due to the diversity and, at the same time, thermodynamics processes interdependence, which significantly complicates the formation of models and determination of deterministic cause-and-effect relationships between the operating conditions of the object of diagnostics and controlled parameters of its condition. The task becomes even more complicated when diagnosing in transient modes of engine operation.
- The developed mathematical model is a set of equations describing the conditions of joint operation of engine elements. The model is designed to determine the parameters of a mixed-flow turbofan engine with an afterburner in steady and transient operating modes. The developed system can be used to adjust the engine and diagnose its operational status during operation.
- A thorough analysis (especially regarding the history of the development of a specific fault) should be carried out on the ground, utilizing the full array of necessary information available to the researcher, both for a particular engine type and for the entire fleet of aviation engines.

- Simplicity, reliability, and acceptable accuracy of the predicted solution should be the prevailing aspirations of the developer of the mathematical model used in the onboard engine control system.

CONFLICT OF INTEREST

Nothing to declare

AUTHORS' CONTRIBUTION

Conceptualization: Kulyk M; **Methodology:** Kulyk M, Lastivka I, and Volianska L; **Software:** Lastivka I, Volianska L, and Babichev I; **Validation:** Kulyk M, Lastivka I, and Volianska L; **Formal analysis:** Kulyk M, Lastivka I, and Volianska L; **Investigation:** Kulyk M, Lastivka I, Volianska L, Voznyuk A, and Babichev I; **Resources:** Lastivka I; **Data Curation:** Kulyk M, Lastivka I, Volianska L, Voznyuk A, and Babichev I; **Writing** - **Original Draft:** Kulyk M, Lastivka I, Volianska L, Voznyuk A, and Babichev I; **Writing** - **Review & Editing:** Volianska L, Voznyuk A, and Babichev I; **Visualization:** - Kulyk M and Lastivka I; **Supervision:** Lastivka I.

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