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A Close Multi-Target Tracking Algorithm Based on Weight Correction

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ABSTRACT

When multiple targets are close to each other and intersect, the Gaussian mixture probability hypothesis density (GM-PHD) filtering algorithm experiences degraded tracking performance. To address this problem, a neighborhood multi-target tracking optimization algorithm based on weight correction is proposed. In the proposed method, a proximity monitoring mechanism is first introduced to detect the distance between targets. Next, the similarity between the measured data and the target predicted value is calculate to form a similarity matrix. If there are multiple data points in a row of the similarity matrix exceed the threshold, further correction should be performed on the data in that row. Finally, the weight correction matrix is formed by combining the above two steps. Simulation results demonstrate that the tracking accuracy and stability of the proposed algorithm are significantly improved in scenarios of multi-target intersection and parallel tracking, and its performance is better than that of the traditional GM-PHD filtering algorithm.

Keywords: Multi-target tracking; GM-PHD; Minimum mean square error matrix; Weight correction.

INTRODUCTION

With the continuous development and improvement of radar, sonar, infrared, laser, and other sensor technologies, target tracking has become a research field that has attracted the attention of many scholars. Its importance in theory and practice has become increasingly prominent, and it has been applied to various fields. Target tracking is the process of using data obtained from sensors in a monitoring system to predict and estimate the position, velocity, acceleration, and other states of a target in real-time or intermittently (Vo *et al.* 2015; Yang *et al.* 2023a; b; Zhang *et al.* 2024). The purpose of this process is to be able to continuously track the trajectory of the target to achieve monitoring, positioning, and tracking of the target.

Early target tracking was mainly a one-to-one approach, i.e., a sensor could only continuously track one target (Ahmad *et al.* 2024). Due to the modern warfare development trend and changing tactics and strategies, single-sensor single-target tracking has gradually failed to meet users' increasingly complex requirements. To adapt to the changing battlefield environment, researchers introduced scan-tracking technology into the tracking systems. This technique can continuously track multiple targets simultaneously while scanning the search space, so that the target tracking problem becomes a problem of multiple targets tracking (MTT) (Wax 1955). There are two main problems in multi-target tracking and the first is the uncertainty of the number of targets. Throughout the tracking process, targets may disappear, be added, or evolve for various reasons, making it difficult to accurately predict and determine the number of targets at each moment. The second is the uncertainty of the measurement data. Since the environment of target tracking changes over time, it is not possible to accurately obtain the environmental parameters at the current moment, which leads to the ambiguity and uncertainty of the correspondence between the measurement data and the target.

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Initially, researchers used a data-based approach to solve these problems, and the results were fruitful. Among these, the representative methods include multiple hypotheses tracking (MHT) (Coraluppi and Carthel 2018) and joint probabilistic data association (JPDA) (He *et al.* 2020). This type of method is mainly based on the correct association of the target and the measurement data. If there is an error in the association of the target and the measurement data, the tracking accuracy will drop sharply. In cases where the number of targets is very small, the correlation is relatively simple. However, as the number of targets increases, the relationship between them becomes intricate. To solve this problem, Mahler (2004) proposed the random finite set (RFS) theory, which models the target state data and the measurement data as two independent random sets, eliminating the need for complex data associations. After the RFS theory was proposed, it has been widely used in the field of multi-target tracking due to its advantages, and a series of filtering algorithms based on RFS have been developed, such as Gaussian mixture probability hypothesis density (GM-PHD) filters (Gning *et al.* 2010; Gunnarsson *et al.* 2007) and sequential Monte Carlo PHD (SMC-PHD) filters (Qin *et al.* 2024). These algorithms are not only computationally intensive, but also do not require data association, which can improve the accuracy of target state and quantity estimation and are easy to implement.

However, with the continuous development of sensor technology and data processing algorithms, the complexity of multi-target tracking systems has become more pronounced. In practical application scenarios, multiple targets are often dense and adjacent to each other, which brings challenges for traditional target tracking algorithms. First, targets that are too close to each other may prevent the sensor from effectively acquiring measurements of these targets. This is due to the sensor's limited resolution and detection range. When the distances between targets are very close, the sensors may not be able to distinguish them accurately, resulting in the loss or confusion of measurement information for some targets. Second, targets in close proximity may lead to multiple measurements corresponding to the same target, which can cause complicates subsequent target tracking and state estimation.

In the multi-target proximity tracking environments, effectively solving the problems of target state estimation, quantity estimation, and data association has become a key research focus. When targets are nearby, two or more measurements may be associated with a single target, violating the one-to-one assumption and causing the performance of the GM-PHD filter to drop sharply. To address this problem Aoki (2016) proposes a multi-target Bayes filter based on the particle labeling method, which describes the uncertainty of target labeling in the iterative process and provides a physical explanation for measuring this uncertainty. Gong and Cui (2022) proposed a spatial proximity multi-target tracking algorithm based on GM-PHD filtering, which improves tracking accuracy by redistributing the weights of the Gaussian components of the target.

In recent years, with advancements in science and technology, several multi-target tracking algorithms have been proposed to address various challenges. For example, to address the multi-target tracking problem in complex environments, Sun *et al.* (2024) proposed a Gaussian mixture probability hypothesis density filter for clutter density estimation; Zhang *et al.* (2023) proposed a maneuvering star convex extended target tracking algorithm based on expected mode augmentation to solve the problem of maneuvering extended target tracking. Tovkach and Zhuk (2021) proposes a sensor network based on received signal strength measurement to solve unknown power anomalies in transmitter measurements, though, it did not consider the relationship between the measured values and the target state. Yan (2014) proposed a Gaussian hybrid PHD tracker based on cooperative punishment for short-range target tracking, which penalizes the weights of targets with the same label by tagging them. However, this algorithm also encounters tracking instability." should be corrected as "Wang (2014) proposed a Gaussian hybrid PHD tracker based on cooperative punishment for short-range target tracking, which penalizes the weights of targets with the same label by tagging them. However, this algorithm also encounters tracking instability.

The main contributions of this paper are as follows:

- A proximity monitoring mechanism is introduced to determine whether targets are in close proximity by calculating the Euclidean distance between them. When the distance between two targets is less than a preset distance threshold, a remediation strategy is triggered.
- The minimum mean square error matrix is introduced. The matrix comprehensively considers multiple historical states and further optimizes tracking accuracy by calculating the weighted mean square error between the historical trajectory and the measurement, thereby more comprehensively reflecting the dynamic characteristics of the target.

- A method for correcting the weights is proposed, which combines the similarity matrix and the minimum mean square error matrix. Based on the calculation of the similarity matrix, the similarity between the measured value and the target state is quantified and further optimized by the minimum mean square error matrix. This synthesis strategy yields a weight correction matrix that adjusts the weights of the GM-PHD filter update step to more accurately reflect the target state.

The rest of this paper is organized as follows: in the problem formulation section, the multi-target RFS modeling, the GM-PHD recursion, and the multi-target proximity problem are presented. The optimization algorithm for close multi-target tracking based on weight correction is proposed in the multi-target weight correction strategy section. In the simulation results and discussions section, simulation results are provided. Finally, conclusions are presented.

Problem formulation

Basic knowledge related to RFS

In multi-target tracking, the number of targets changes at any given moment. This is because, at any given moment, the target will disappear, survive, or spawn (Mahler 2019). The emergence of RFS theory can better describe this evolutionary process. RFS defines the state and measurements of multiple targets as RFS variables, respectively, where each set represents a set of targets or measurements. In this way, it is possible to better characterize the dynamic changes of targets and the uncertainty of sensor measurements in multi-target systems (Mahler 2003, 2009a; b). The details are as follows:

$$X_k = \left\{x_{k,1}, x_{k,2}, \ldots, x_{k,N_k}\right\} \tag{1}$$

$$Z_k = \left\{z_{k,1}, z_{k,2}, \ldots, z_{k,M_k}\right\} \tag{2}$$

where X_k is the set of target states at time k, Z_k is the set of measurements at time k, N_k is the number of targets at time k, M_k is the number of measurements at time k $x_{k,i}$ ($i = 1, 2, \ldots, N_k$) is the state of the i th target at time k, and $z_{k,j}$ ($j = 1, 2, \ldots, M_k$) is the jth measurement at time k.

If X_{k-1} is used to represent the set of multi-target states at time $k-1$, considering the multiple situations that exist for the target at each time, the set of multi-target states at time k can be expressed as follows:

$$X_k = \left\{\bigcup_{x_{k-1}\in X_{k-1}} B_{k|k-1}(x_{k-1})\right\} \cup \left\{\bigcup_{x_{k-1}\in X_{k-1}} S_{k|k-1}(x_{k-1})\right\} \cup \Gamma_k \tag{3}$$

where $B_{k|k-1}(x_{k-1})$ and $S_{k|k-1}(x_{k-1})$ respectively represent the RFS of states of the x_{k-1}-derived target and the survival target at time k and Γ_k represents the new target set at time k.

On the one hand, the number of measurements at each moment is uncertain and disordered, and on the other hand, targets may either be detected or missed by sensors. Therefore, it can be assumed that each state xk ∈ X_k produces an RFS $\Theta_k(x)$, where $\Theta_k(x)$ is $\{z_k\}$ when the target is detected and $\Theta_k(x)$ is an empty set when the target is not detected. However, there is clutter interference in the actual environment, resulting in the measurement set obtained by the sensor containing not only the measurement of the real target but also includes the set of false alarms or clutter.

Then the model established using the stochastic finite set for the observation of multi-target at time k is as follows:

$$Z_k = K_k \cup \left[\bigcup_{x\in X_k} \Theta_k(x)\right] \tag{4}$$

where K_k is a collection of clutter.

According to the probabilistic and statistical properties of the stochastic finite set, a PHD filter is proposed. The filter uses the sensor at time k to measure RFS Z_k, and iteratively transmits the intensity function of the multi-point target at time k through the prediction step and the update step, in order to realize multi-target tracking. The specific filtering steps are as follows:

Prediction step

Assuming that the multi-target state intensity function at time $k-1$ is $D_{k-1|k-1}(X_{k-1}|Z_{1:k-1})$, the probability that single-target state X_{k-1} will continue to exist at time k is $P_{S,k|k-1}(X_{k-1})$, and the transition from target state X_{k-1} to target state x_k conforms to Markovality, which can be described by the state transition probability density function $f_{k|k-1}(x_k|x_{k-1})$. In addition, the usable state transition probability density function $\beta_{k|k-1}(x_k|x_{k-1})$ of the derived target process generated by the single-point target state x_{k-1} at time k is described, and the RFS intensity function of the nascent target is $\gamma_k(x_k)$.

According to the generalized finite set statistics (FISST) theory, the prediction formula is:

$$D_{k|k-1}(x_k|Z_{1:k-1}) = \gamma_k(x_k) + \int \Phi_{k|k-1}(x_k|x_{k-1}) D_{k-1|k-1}(x_{k-1}|Z_{1:k-1}) dx_{k-1} \tag{5}$$

$$\Phi_{k|k-1}(x_k|x_{k-1}) = p_{S,k|k-1}(x_{k-1}) f_{k|k-1}(x_k|x_{k-1}) + \beta_{k|k-1}(x_k|x_{k-1}) \tag{6}$$

In the above equation, $D_{k-1|k-1}(X_{k-1}|Z_{1:k-1})$ denotes the predicted intensity function of the multi-target state.

Update step

Assuming that the measurement set at time k is RFS Z_k and the probability of a single point target X_k being detected by the sensor is $P_{D,k}(x_k)$, the measurement process can be described by the point target likelihood function $f(z_k|z_k)$, the measurement of each target is independent of each other, and the target measurement and clutter RFS are independent of each other, then the formula for updating the prediction intensity function by using the measurement data at the current time is:

$$D_{k|k}(x_k|Z_{1:k}) = \left(1 - p_{D,k}(x_k)\right) D_{k|k-1}(x_k|Z_{1:k-1}) + \sum_{z_k \in Z_k} \frac{p_{D,k}(x_k) f(z_k|x_k) D_{k|k-1}(x_k|Z_{1:k-1})}{K_k(z_k) + \int p_{D,k}(x_k) f(z_k|x_k) D_{k|k-1}(x_k|Z_{1:k-1}) dx_k} \tag{7}$$

In the above equation, $D_{k|k}(X_k|Z_{1:k})$ is the posterior intensity function of multi-target RFS at time k and $K_k(Z_k)$ is the intensity function of clutter. Assuming that the number of clutters obeys a uniform distribution with a Poisson ratio of λ_k and the probability density function of the spatial distribution of clutter is $c_k(Z_k)$, then the formula for calculating $K_k(z_k)$ is:

$$K_k(z_k) = \lambda_k c_k(z_k) \tag{8}$$

The GM-PHD recursion

By studying the recursive formula of the PHD filter, it becomes apparent that it involves complex operations of multiple integrals in the prediction and update processes, making it difficult to obtain an analytical solution. To address this, three assumptions are proposed in (Clark *et al.* 2006) under linear Gaussian conditions, providing an analytical solution for the recursive Eqs. 5 and 7 of PHD filters.

The three assumptions for the GM-PHD filter are as follows:

- It is assumed that the motion model and measurement model of each target are linear Gaussian:

$$f_{k|k-1} = (x|\xi) = N(x; F_{k-1}\xi, Q_{k-1}) \tag{9}$$

$$g_k(z|x) = N(z; H_k x, R_k) \tag{10}$$

where $N(; m,P)$ is the density of Gaussian probability assumptions with a mean of m and a covariance of P, F_{k-1} is the state transition matrix, ξ is the posterior of the state vector x in the previous time step k-1, H_k is a measurement matrix, and R_k is the covariance matrix of the noise.

- It is assumed that the survival probability and the detected probability of the target are independent of the target state:

$$p_{S,k}(x) = p_{S,k} \tag{11}$$

$$p_{D,k}(x) = p_{D,k} \tag{12}$$

where $P_{S,k}$ represents the target survival probability and $P_{D,k}$ is the detection probability.

- The intensity of the new target RFS is assumed to be in Gaussian mixture:

$$B_k(x) = \sum_{i=1}^{J_{B,k}} w_{B,k}^{(i)} N\left(x; m_{B,k}^{(i)}, P_{B,k}^{(i)}\right) \tag{13}$$

where $w_{B,k}^{(i)}$, $m_{B,k}^{(i)}$, and $P_{B,k}^{(i)}$ are, respectively, the weight, mean, and covariance of the ith new target at time k and $J_{B,k}$ is the number of new target at time k.

Based on these three assumptions, the PHD filter can obtain the following analytical solution.

The Gaussian mixture of the predicted intensity function is as follows:

$$v_{k|k-1}(x) = \sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^{(i)} N\left(x; m_{k|k-1}^{(i)}, P_{k|k-1}^{(i)}\right) \tag{14}$$

where $J_{k|k-1}$ represents the number of predicted Gaussian components, $w_{B,k-1}^{(i)}$, $m_{B,k-1}^{(i)}$ and $P_{B,k-1}^{(i)}$ represent the weight, mean, and covariance of the ith predicted Gaussian components, respectively.

Depending on whether the target is detected by the sensor or not, the formula for calculating the posterior intensity function $v_{k|k}(x)$ (i.e., updating the formula) is as follows:

$$v_{k|k}(x) = \left(1 - p_{D,k}\right) v_{k|k-1}(x) + \sum_{z_k \in Z_k} v_{D,k}(x|z_k) \tag{15}$$

where

$$v_{D,k}(x|z_k) = \sum_{j=1}^{J_{k|k-1}} w_{D,k|k}^{(j)} N\left(x; m_{D,k|k}^{(j)}, P_{D,k|k}^{(j)}\right) \tag{16}$$

$$w_{D,k|k}^{(j)} = \sum_{z_k \in Z_k} \frac{p_{D,k} w_{k|k-1}^{(j)} N\left(z_k; H_k m_{k|k-1}^{(j)}, R_k + H_k P_{k|k-1}^{(j)} H_k^T\right)}{K_k(z_k) + p_{D,k} \sum_{l=1}^{J_{k||k-1}} w_{k|k-1}^{(l)} N\left(z_k; H_k m_{k|k-1}^{(l)}, R_k + H_k P_{k|k-1}^{(l)} H_k^T\right)} \tag{17}$$

$$m_{D,k|k}^{(j)} = m_{k|k-1}^{(j)} + K_k^{(j)}(z_k - H_k m_{k|k-1}^{(j)}) \tag{18}$$

$$P_{D,k|k}^{(j)} = \left[I - K_k^{(j)} H_k\right] P_{k|k-1}^{(j)} \tag{19}$$

$$K_k^{(j)} = P_{k|k-1}^{(j)} H_k^T \left(H_k P_{k|k-1}^{(j)} H_k^T + R_k\right)^{-1} \tag{20}$$

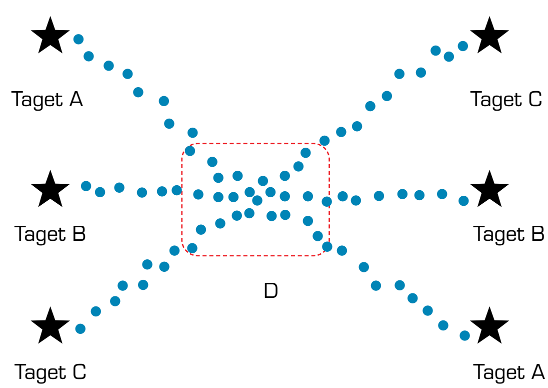
Multi-target proximity problem

In a point target tracking system, it is generally assumed that there is a unique correspondence between each target and its measurement data. At any discrete moment, a maximum of one measurement value can be generated per target, and each measurement can only be associated with one target. Compared with other methods, the GM-PHD algorithm does not require a strict one-to-one relationship between the target and the measurement. It implicitly considers the association between all possible targets and the measurement data, assuming that the prior probability that each measurement data is associated with a different target is the same. Therefore, in the GM-PHD algorithm, one measurement data can match multiple targets, and there may be multiple measurement data matching the same target. After the cropping and merging process, those parts that weigh more than the threshold are extracted as the target estimates.

When targets are far apart, there is a significant difference in the weight of the Gaussian components generated by different targets, so the filter can more accurately determine which target the measurement data matches better. In this case, the filter is able to determine exactly which target the measurement data is coming from because there is clear discrimination. However, when multiple targets are in close proximity to each other, the difference in the weights of the Gaussian components becomes less obvious. As a result, the merging of different targets in the Gaussian component merging stage leads to a decrease in the number of target estimates, which affects the performance of the multi-target tracking system.

In the case of multiple targets close to each other, the GM-PHD algorithm may experience performance degradation because the relationship between the measurement data and the target cannot be accurately determined. When multiple targets are close to each other, the weight relationship between the measurement data and the target is ambiguous, which brings challenges to the tracking system.

Figure 1 shows a multi-target cross-tracking measurement distribution scenario. Within region D, target A, target B, and target C are in close proximity, causing their measurement data to be mixed. This situation can cause the target forecast strength to be incorrectly updated during the update phase. In the GM-PHD filter, when n targets cross or approach, ideally, the PHD distribution at the intersection location should have n peaks, with each peak representing a target. However, the GM-PHD filtering algorithm does not limit the one-to-one relationship between the target and the measurement. A target can be disturbed by measurements from adjacent targets, causing its peaks to split into multiple peaks with less intensity, i.e., the Gaussian component with less weight. In the merging and clipping stage, it is difficult to recombine these components into the correct number of targets, which leads to inaccurate estimation of the number of targets and reduces the estimation accuracy of the target state.



Source: Elaborated by the authors.

Figure 1. Multi-target cross tracking.

Multi-target weight correction strategy

When the GM-PHD filter copes with multi-target tracking scenarios involving close parallel motion and staggered motion, the motion trajectories in the sensor's field of view are very similar, resulting in a lack of discrimination in the measurement information received by the sensor. In this case, it may be difficult for the filter to accurately distinguish between measurements of different targets, leading to mismatches between measurements and targets.

Proximity monitoring mechanism

First, a monitoring mechanism is introduced to determine when targets are in proximity. If multiple targets are found to be in close proximity, the corresponding remediation strategy is executed.

Assuming that the predicted mean of the ith Gaussian component $m^{(i)}_{k,k-1}$ is $(x^i_{k-1|k}, y^i_{k-1|k})$ and the predicted mean of the jth Gaussian component $m^{(i)}_{k,k-1}$ is $(x^j_{k-1|k}, y^j_{k-1|k})$ at time k, the distance between the two is:

$$d_{ij} = \sqrt{\left(x^i_{k-1|k} - x^j_{k-1|k}\right)^2 + \left(y^i_{k-1|k} - y^j_{k-1|k}\right)^2} \tag{21}$$

$$d_{ij} < \delta \tag{22}$$

where δ is the distance threshold.

By calculating the distance relationship between multiple targets and setting a distance threshold δ, it is possible to determine whether the targets are in a close state. If the distance between the targets is less than the threshold, you can assume that they are moving closer to each other and begin to execute the following remediation strategy.

Similarity matrix

Suppose that the predicted mean of the ith Gaussian component $m^{(i)}_{k|k-1}$ is $(x^i_{k-1|k}, y^i_{k-1|k})$ and the measurement set is $Z_k = \{Z^1_k, \ldots Z^j_k\}$ at time k. Calculate the residual vector between each measured value and each target state:

$$\varepsilon^{ij}_k = z^j_k - H_k m^i_{k|k-1} \tag{23}$$

Using the residual vector and the covariance matrix, the Mahalanobis distance matrix can be calculated; for each observation to the target state, the Mahalanobis distance is calculated by the following formula:

$$D^{ij}_k = \sqrt{(\varepsilon^{ij}_k)^T P^{i\,-1}_k \varepsilon^{ij}_k} \tag{24}$$

Each element in the Mahalanobis distance matrix is converted to a similarity value to construct a similarity matrix. In general, you can use the reciprocal of the distance as the similarity value, i.e., the similarity is:

$$similarity_{ij} = \frac{1}{D_{ij}} \tag{25}$$

This results in a $i \times j$ similarity matrix of $\Re_k$:

$$\Re_k = \begin{bmatrix} similarity_{11} & \cdots & similarity_{1j} \\ \vdots & \ddots & \vdots \\ similarity_{i1} & \cdots & similarity_{ij} \end{bmatrix} \tag{26}$$

By looking at the values in the similarity matrix, it is possible to determine which measurements have a high degree of similarity to which target states. Typically, a similarity threshold can be set to determine whether the similarity has reached a certain level, thus determining the membership of the measurement. A larger similarity value indicates a higher degree of correlation between the measurement and the target state, while a smaller value indicates a lower degree of association.

Minimum mean square error matrix

In some complex tracking scenarios, the similarity matrix may show that the target and the measurement may be one-to-many or many-to-one, which can be judged by whether there are multiple data points in each row of the similarity matrix that are greater than the similarity threshold. In order to further optimize, only the data with this problem are corrected by constructing the weighted fusion of the minimum mean square error between the historical trajectory of the target and the measurement, to achieve accurate tracking of the motion of adjacent multiple targets.

The root mean square of the minimum error between the target state estimation from time $k - n$ to time k and the measurement of the current time k is calculated separately, and the matrix formed by weighted fusion is as follows:

$$M_k = \begin{bmatrix} G^{1,1}A^T & \cdots & G^{1,j}A^T \\ \vdots & \ddots & \vdots \\ G^{i,1}A^T & \cdots & G^{i,j}A^T \end{bmatrix} \tag{27}$$

where;

$$G^{i,j} = \left[\varepsilon_{k-n}^{n}(m_{k-n}^{i}, z_k^j), \dots, \varepsilon_{k-(\varphi-1)}^{0}(m_k^i, z_k^j)\right]_{1\times\varphi}, \forall n = 1{:}\,(\varphi-1) \tag{28}$$

$$\varepsilon_k^{ij} = \left(z_k^j - H_k m_{k|k-1}^i\right)^2 \tag{29}$$

$$\varepsilon_{k-n}^{n} = \left(z_k^j - H_{k-n} m_{k-n}^i\right)^2 \tag{30}$$

$$A = [1, (\varphi-1)/\varphi, \dots, 1/\varphi] \tag{31}$$

In the above equation, ε_{k-n}^{n} (m_{k-n}^{i}, Z_k^j) is the estimation of the minimum mean square error of the state estimation of the *j*th observation data z_k^j at time *k* and the *i*th target at time *k* – *n*, and the different values from 1 to 6 of *n* are verified by simulation experiments. Comparing the effects of selecting different first *n* moments, it is found that the effect of taking 5 from *n* is better.

Therefore, it is chosen to fuse the root mean square weighting of the target state estimation at the first five moments with the minimum error of the current measurement calculation.

Weight correction matrix

Based on the similarity matrix and the minimum mean square error matrix, the matrix Λ_k used to correct the target update step is:

$$\Lambda_k = \begin{bmatrix} \alpha^{1,1} & \cdots & \alpha^{1,j} \\ \vdots & \ddots & \vdots \\ \alpha^{i,1} & \cdots & \alpha^{i,j} \end{bmatrix} \tag{32}$$

$$\alpha_{ij} = \begin{cases} 1, \underset{j}{argmin}\left(M_k(i,j)\right), \forall i = 1{:}\,J_{k|k-1}, \forall j = 1{:}\,Z_k \\ similarity_{ij}, otherwise \end{cases} \tag{33}$$

Then, based on the sensor measurement set at time *k*, the target prediction intensity and the correction matrix, the target posterior intensity can be expressed as:

$$v_{k|k}(x) = \left(1 - p_{D,k}\right) v_{k|k-1}(x) + \textstyle\sum_{z\in Z_k^t} v_{D,k}(x|z_k) \tag{34}$$

$$v_{D,k}(x|z_k) = \textstyle\sum_{n=1}^{J_{k|k-1}} w_{k|k}^{n,j} N\left(x_k; m_{D,k|k}^{n,j}\left(z_k^j\right), P_{D,k|k}^j\right) \tag{35}$$

$$w_{k|k}^{n,j} = \frac{\alpha_k^{n,j} p_{D,k} w_{k|k-1}^j N\left(\left(z_k; H_k m_{k|k-1}^j\right), R_k + H_k P_{k|k-1}^j H_k^T\right)}{c_k^i(z) + p_{D,k} \sum_{i=1}^{J_{k|k=1}} \alpha_k^{i,j} w_{k|k-1}^j N\left(\left(z_k; H_k m_{k|k-1}^j\right), R_k + H_k P_{k|k-1}^j H_k^T\right)} \tag{36}$$

Weight Correction -Gaussian mixture probability hypothesis density (GM-PHD) filtering

A correction strategy is presented to handle the multi-target proximity problem. First, the monitoring mechanism determines when the targets are approaching, and if multiple targets are found to be approaching each other, corresponding correction strategies are executed. In the calibration process, the correlation between data features is comprehensively considered. Simultaneously, in order to avoid a one-to-one situation between the target and the measurement, the minimum mean square error is calculated between the historical state of the target at the previous time and the measurement at the current time. Considering the similarity between the target state and the current measurement information, as well as the minimum mean square error between the historical state of the target and the measurement value, a weight correction matrix is formed. The flowchart of this algorithm is shown in Table 1.

Table 1. The flowchart of the proposed filtering method.

Step	Title	Description
1	Proximity monitoring mechanism	a) Calculate the Euclidean distance d_{ij} between targets; b) Determine $d_{ij} < \delta$ to determine whether there is a neighbor target.
2	Similarity matrix	a) Calculate the residual vector ε_{ij}; b) Calculate the Mahalanobis distance D_{ij}; c) Calculate the similarity matrix $\Re_k$.
3	Minimum mean square error matrix	a) Check whether there are multiple data in each row of the similarity matrix that is greater than the threshold λ; b) Construct the minimum mean square error between the historical state of the target and the measurement.
4	Weight correction matrix	Based on the similarity matrix and the minimum mean square error matrix, the weight correction matrix was calculated Λ_k.
5	Update	Use Λ_k for the weight w_k^i correction of the update step.

Source: Elaborated by the authors.

SIMULATION RESULTS AND DISCUSSIONS

To verify the effectiveness of the proposed weight correction algorithm, it was compared it with the GM-PHD filter in different scenarios, such as multi-target intersecting motion and multi-target parallel motion. These scenarios help evaluate the performance of the algorithm in response to different situations and target proximity tracking challenges.

In these two scenarios, the target detection probability is $P_d = 0.98$, the target survival probability is $P_s = 0.98$, the Gaussian weight clipping threshold is $T_p = 10^{-5}$, the merge threshold is $T_m = 5$, and the maximum number of Gaussian components is J_{max}.

The above simulation scenarios were simulated on a PC with Intel(R) Core(TM) i5-10400 CPU @ 2.90GHz processor and 8G memory, using MATLAB R2020a.

A tracking scene where multiple target trajectories intersect

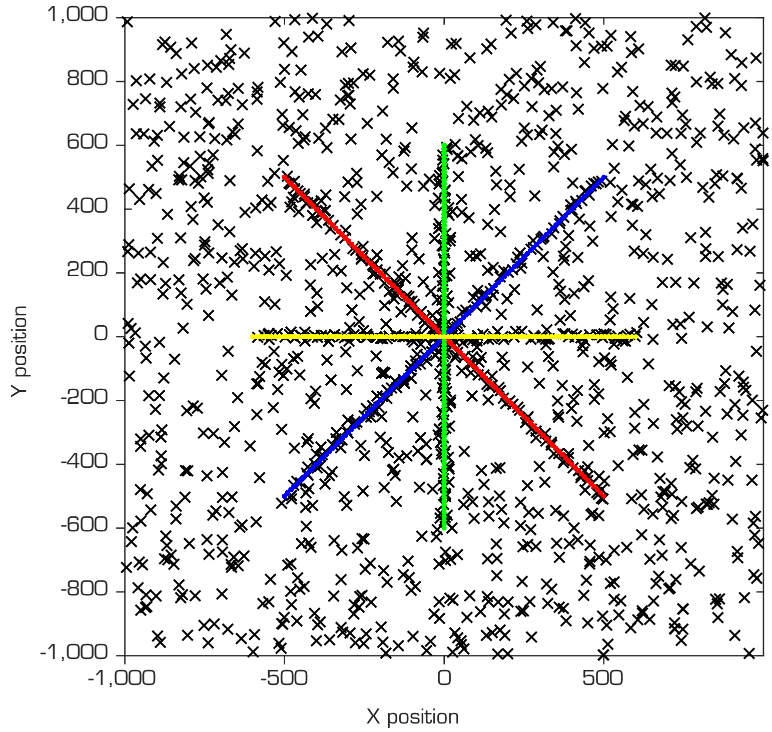
In this scenario, the estimation of the four crossover targets over the entire observation time and the measurement of clutter are shown in Fig. 2.

The total duration of the real motion trajectory simulation experiment is k = 20 s, and four targets appear at k = 1 s and disappear at k = 20 s. These targets approach each other from different positions, crossing at the origin and then separating. The initial intensity function of the multi-target is:

$$\begin{aligned} D_b(x) &= 0.1N\left(x; m_b^{(1)}, P_b\right) + 0.1N\left(x; m_b^{(2)}, P_b\right) \\ &+ N\left(x; m_b^{(3)}, P_b\right) + 0.1N\left(x; m_b^{(4)}, P_b\right) \end{aligned} \quad (37)$$

where

$$m_b^{(1)} = [-500 \quad 500 \quad 0 \quad 0]^T, m_b^{(2)} = [-500 \quad -500 \quad 0 \quad 0]^T,$$
$$m_b^{(3)} = [0 \quad 600 \quad 0 \quad 0]^T,\ m_b^{(4)} = [-600 \quad 0 \quad 0 \quad 0]^T, \tag{38}$$
$$P_b = \mathrm{diag}([100 \quad 100 \quad 25 \quad 25])$$



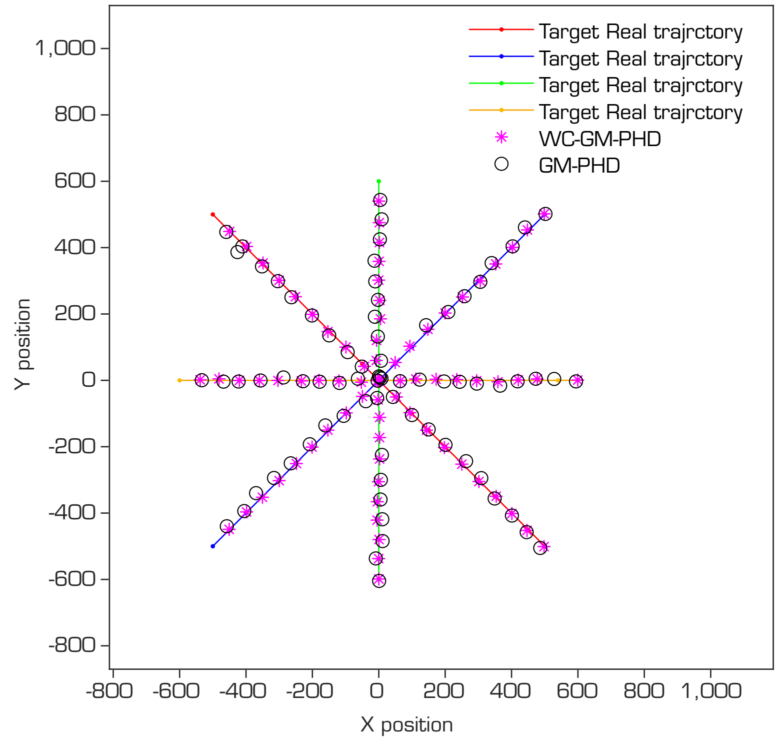
Source: Elaborated by the authors.

Figure 2. The true trajectories of targets and measurements with clutters in the tracking scenario with crossed trajectories.

In order to verify the effectiveness of the adjacent multi-target tracking algorithm based on the comprehensive similarity weighting method of the target's historical state, the proposed algorithm is compared with the ordinary GM-PHD algorithm in the multi-target intersection scenario. Figures 3 and 4 show the comparison results of the target trajectories and optimal sub-pattern assignment (OSPA) distance results of the two algorithms.

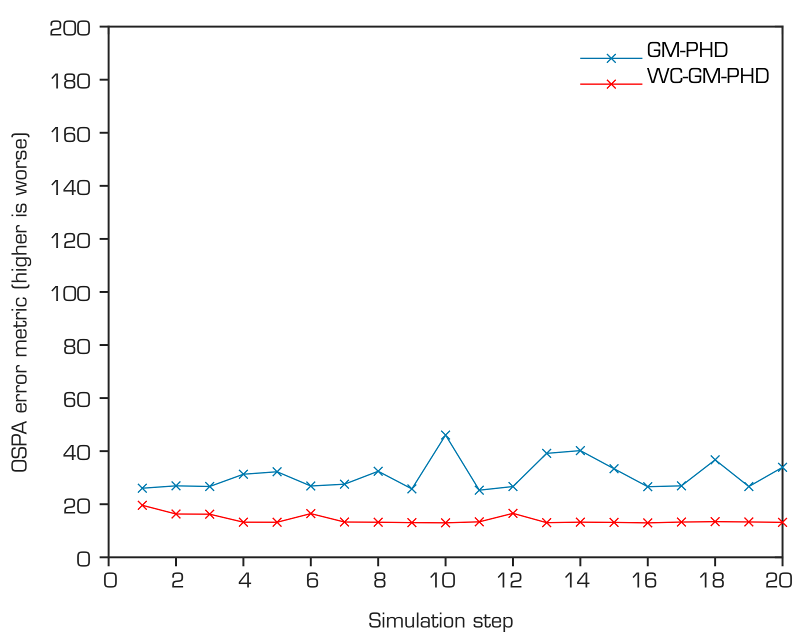
It can be seen from Fig. 3 that when the target moves in a straight line at a uniform speed and is far apart, the two algorithms can track the target stably. However, when the targets are close at the moment of crossing, the target state estimated by the GM-PHD filter deviates from the correct target trajectory, and the blue and green targets even experience multiple moments of tracking loss. The target tracking trajectory of the algorithm in this paper, at the same time, is not only basically covered by the real trajectory, but also has a higher tracking accuracy than the GM-PHD filter. This shows that the proposed algorithm can accurately distinguish and track multiple targets when the target distance is close, and the effect is obviously better than that of the GM-PHD filter algorithm. Due to the comprehensive similarity weighting method of multi-target historical state measurement, the algorithm can estimate the similarity between targets more accurately, thereby better maintaining the tracking state. Table 2 shows an overall OSPA metric in the simulation.

One hundred Monte Carlo experiments were also performed with detailed statistical analysis of the performance of both algorithms. As can be seen in Fig. 4, the OSPA error is similar for both algorithms when there is no target crossing. However, there is a significant difference in the performance of the two algorithms during target proximity or crossing. In this case, the GM-PHD algorithm shows a large OSPA error, while the WC-GM-PHD exhibits a small error, comparable to the OSPA error when there is



Source: Elaborated by the authors.

Figure 3. Multi-target tracking trajectories of two approaches.



Source: Elaborated by the authors.

Figure 4. Tracking errors of the two approaches.

Table 2. Mean comparison of OSPA in scenario A.

GM-PHD	WC-GM-PHD
30.874	14.162

Source: Elaborated by the authors.

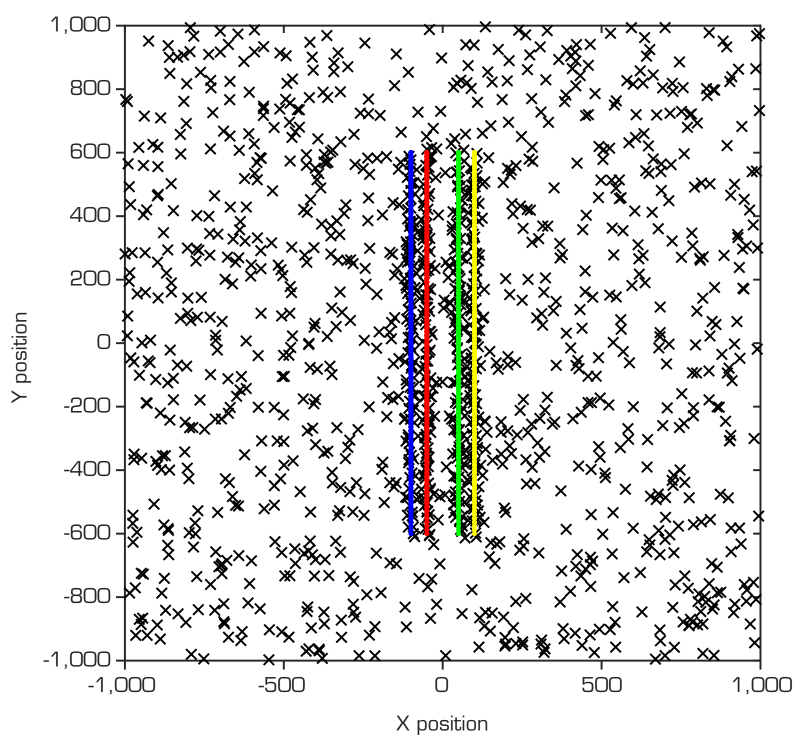
no crossover. This result highlights the superiority of the algorithm with target intersection during target crossing. It can be seen that by using the historical state of the target to obtain the optimal match, the algorithm can obtain a more accurate target state estimation result during the adjacent period of multiple target intersections.

A tracking scene where multiple target trajectories are parallel

In this scenario, the estimation of the four targets approaching in parallel over the entire observation time and the measurement of the clutter are shown in Fig. 5.

The total time of the simulation experiment is k = 50 s. Four targets appear at k = 1 s and disappear at k = 50 s, moving in parallel. The initial intensity function of the multi-target is:

$$D_b(x) = 0.1N\left(x; m_b^{(1)}, P_b\right) + 0.1N\left(x; m_b^{(2)}, P_b\right) + N\left(x; m_b^{(3)}, P_b\right) + 0.1N\left(x; m_b^{(4)}, P_b\right) \quad (39)$$



Source: Elaborated by the authors.

Figure 5. The true trajectories of targets and measurements with clutter in the tracking scenario with crossed trajectories.

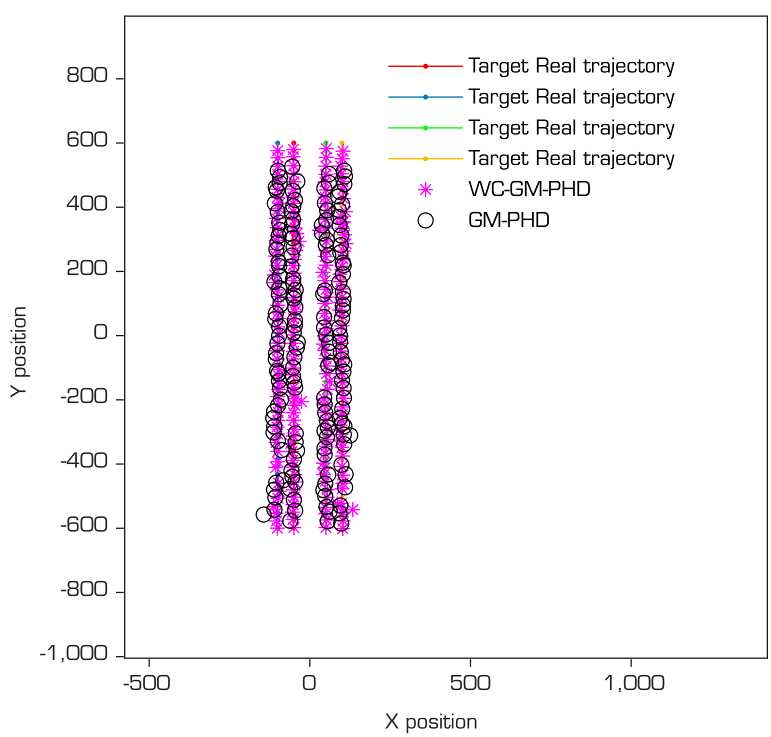
where

$$\begin{aligned} m_b^{(1)} &= [-50 \quad 600 \quad 0 \quad 0]^T, m_b^{(2)} = [-100 \quad 600 \quad 0 \quad 0]^T, \\ m_b^{(3)} &= [50 \quad 600 \quad 0 \quad 0]^T, m_b^{(4)} = [100 \quad 600 \quad 0 \quad 0]^T, \\ P_b &= \text{diag}([100 \quad 100 \quad 25 \quad 25]) \end{aligned} \tag{40}$$

Similarly, experimental simulations were conducted in a multi-target parallel proximity tracking scenario. Figures 6 and 7 show the trajectory comparison and OSPA distance of the two algorithms.

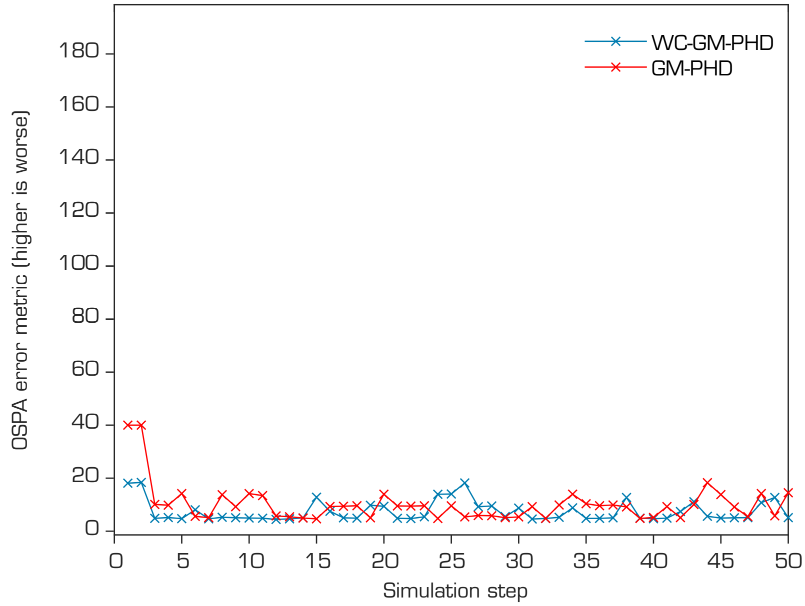
From the trajectory comparison in Fig. 6, it can be observed that the GM-PHD algorithm has some shortcomings in dealing with the immediate target tracking, primarily manifested in the loss of target tracking at multiple moments. The target tracking results of the algorithm proposed in this paper are accurately overlaid on the real trajectory. When the distance between two targets is large, the maximum target posterior intensity of the GM-PHD filter is distributed around the estimated state of their respective targets, and they are not affected by each other. However, if the distance between two targets is too close (proximity state), the target posterior strength will only output a single one, with its maximum value existing near the mean of the estimated state of the two targets. Furthermore, in a tracking environment with multiple adjacent targets, the GM-PHD filtering algorithm tends to make error in estimating the target's state. In contrast, the proposed algorithm optimizes the weight allocation of the target by introducing a correction matrix, thereby improving the performance defects of the GM-PHD algorithm in proximity target tracking.

Further observation of the data in Fig. 7 shows that the OSPA performance index of the improved algorithm is significantly better than that of the GM-PHD filtering algorithm, indicating that the improved algorithm has achieved a higher level of target state estimation accuracy. This result highlights the effectiveness and superiority of the algorithm proposed in this paper in dealing with proximity target tracking.



Source: Elaborated by the authors.

Figure 6. Multi-target tracking trajectories of two approaches.



Source: Elaborated by the authors.

Figure 7. Tracking errors of the two approaches.

Table 3 shows an overall OSPA metric in the simulation.

Table 3. Mean comparison of OSPA in scenario B

GM-PHD	WC-GM-PHD
10.132	7.544

Source: Elaborated by the authors.

In interpreting these results, it can be noted that the advantage of the algorithm in this paper lies in its design during the filtering update stage. The algorithm makes full use of the historical state estimation of each target, enabling it to effectively match the measurement closest to the real state of each target from the measurement set at each discrete time. The algorithm minimizes the problem of false state updates that can occur at the intersection of targets.

CONCLUSION

In the field of multi-target tracking, it is critical to determine the correct correlation between each measurement and the target state. To achieve this, the Mahalanobis distance is introduced as a distance measure that considers the correlation between data features, allowing for accurately measurement of the similarity between the measurement and the target state. Additionally, to avoid the many-to-one correlation problem, the minimum mean square error matrix between the historical state of the target from the past moment to the current moment and the measurement is calculate, and, finally, a correction matrix is constructed to adjust the target weight in the update step. This approach makes full use of historical state information and combines it with measurement data from the current moment in time, making the correlation results more reliable and accurate. By considering the minimum mean square error between the historical state and the measurement, the measurement most consistent with the target can be effectively identified, avoiding the many-to-one correlation problem, and improving the robustness and stability of the tracking system.

CONFLICT OF INTEREST

Nothing to declare.

AUTHORS' CONTRIBUTION

Conceptualization: Lifan Sun; **Data curation:** Liyang Xu and Wenhui Xue; **Acquisition of funding:** Lifan Sun; **Research:** Liyang Xu and Wenhui Xue; **Methodology:** Lifan Sun; **Investigation:** Wenhui Xue; **Resources:** Lifan Sun and Wenhui Xue; **Supervision:** Lifan Sun and Wenhui Xue; **Writing - Preparation of original draft:** Lifan Sun and Liyang Xu; **Writing - Proofreading and editing:** Lifan Sun and Liyang Xu"

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Data sharing is not applicable.

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REFERENCES

Ahmad BI, Harman S, Godsill S (2024) A Bayesian track management scheme for improved multi-target tracking and classification in drone surveillance radar. IET Radar Sonar Navig 18(1):137-146. https://doi.org/10.1049/rsn2.12458

Aoki EH, Mandal PK, Svensson L, Bores Y, Bagchi A (2016) Labeling uncertainty in multitarget tracking. IEEE Trans Aerosp Electron Syst 52(3):1006-1020. https://doi.org/10.1109/TAES.2016.140613

Clark DE, Panta K, Vo BN (2006) The GM-PHD filter multiple target tracker. Paper presented 2006 9th International Conference on Information Fusion. IEEE Florence, Italy. https://doi.org/10.1109/ICIF.2006.301809

Coraluppi SP, Carthel CA (2018) Multiple-hypothesis tracking for targets producing multiple measurements. IEEE Trans Aerosp Electron Syst 54(3):1485-1498. https://doi.org/10.1109/TAES.2018.2796478

Gning A, Mihaylova L, Maskell S, Pang S, Godsill S (2010) Group object structure and state estimation with evolving networks and Monte Carlo methods. IEEE Trans Signal Process 59(4):1383-1396. https://doi.org/10.1109/TSP.2010.2103062

Gong Y, Cui C (2022) Spatial proximity multi target tracking algorithm based on GM-PHD filtering. Syst Eng Electron 44(1):76-85" should be corrected as "Gong Y, Cui C (2022) Spatial proximity multi target tracking algorithm based on GM-PHD filtering. Syst Eng Electron 44(1):76-85. https://link.cnki.net/urlid/11.2422.TN. 20210830.1520.009

Gunnarsson J, Svensson L, Danielsson L, Bengtsson F (2007) Tracking vehicles using radar detections. Paper presented 2007 IEEE Intelligent Vehicles Symposium. IEEE; Istanbul, Turkey. https://doi.org/10.1109/IVS.2007.4290130

He S, Shin HS, Tsourdos A (2020) Information-theoretic joint probabilistic data association filter. IEEE Trans Autom Control 66(3):1262-1269. https://doi.org/10.1109/TAC.2020.2989766

Mahler R (2004) Random sets: unification and computation for information fusion-a retrospective assessment. Paper presented 2004 Proceedings of the Seventh International Conference on Information Fusion. IEEE; Stockholm, Sweden. https://www.researchgate.net/publication/228800315

Mahler R (2009a) CPHD and PHD filters for unknown backgrounds I: dynamic data clustering. Sensors and Systems for Space Applications III. International Society for Optics and Photonics. https://doi.org/10.1117/12.818022

Mahler R (2009b) CPHD and PHD filters for unknown backgrounds II: multitarget filtering in dynamic clutter. Sensors and systems for space applications III. International Society for Optics and Photonics. https://doi.org/10.1117/12.818023

Mahler R (2019) Exact closed-form multitarget Bayes filters. Sensors 19(12):2818. https://doi.org/10.3390/s19122818

Mahler RPS (2003) Multitarget Bayes filtering via first-order multitarget moments. IEEE Trans Aerosp Electron Syst 39(4):1152-1178. https://doi.org/10.1109/TAES.2003.1261119

Qin Y, Han Y, Li S, Li J(2024) An iteratively extended target tracking by using decorrelated unbiased conversion of nonlinear measurements. Sensors 24(5):1362. https://doi.org/10.3390/s24051362

Sun L, Xue W, Gao D (2024) Modified Gaussian mixture probability hypothesis density filtering using clutter density estimation for multiple target tracking. J Aerosp Technol Manag 16:e624. https://doi.org/10.1590/jatm.v16.1325

Tovkach IO, Zhuk SY (2021) Filtration of UAV movement parameters based on the received signal strength measurement sensor networks in the presence of anomalous measurements of unknown power at the transmitter. J Aerosp Technol Manag 13:e0921. https://doi.org/10.1590/jatm.v13.1191

Vo B, Mallick M, Bar-Shalom Y (2015) Multitarget tracking. Wiley Encyclopedia of Electrical and Electronics Engineering. city; publisher. https://doi.org/10.1002/047134608x.w8275

Wang Y, Meng H, Liu Y, Wang X (2014) Collaborative penalized Gaussian mixture PHD tracker for close target tracking. Guilin, Guangxi, China; Signal Process 102:1-15. https://doi.org/10.1016/j.sigpro.2014.01.034

Wax N (1955) Signal-to-noise improvement and the statistics of track populations. J Appl Phys 26(5):586-595. https://doi.org/10.1063/1.1722046

Yang C, Cao X, Shi Z (2023a) Road-map aided Gaussian mixture labeled multi-Bernoulli filter for ground multi-target tracking. IEEE Trans Veh Technol 72(6):7137-7147. https://doi.org/10.1109/TVT.2023.3240740

Yang J, Xu M, Li F, et al. (2023b) Adaptive tracking and classification algorithm for multiple extended targets based on irregular shape driven. Island of Rhodes, Greece; Digit Signal Process 136:103992. https://doi.org/10.1016/j.dsp.2023.103992

Zhang J, Sun L, Gao D (2023) Maneuvering star-convex extended target tracking based on modified expected mode augmentation algorithm. J Aerosp Technol Manag 15:e2323. https://doi.org/10.1590/jatm.v15.1314

Zhang J, Zeng C, Tao H, et al. (2024) An adaptive track initiation method adopting multi-scan spatiotemporal information. Vienna, Austria; IEEE Sens J 24(4):4722-4734. https://doi.org/10.1109/JSEN.2023.3344491