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Fixed-Time Delay Calculation Method Based on Fuze-Warhead Coordination: Approach Cases and Application to The Small Target Platform

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ABSTRACT

This paper presents a method for calculating the delay time and determining the fixed delay components for laser fuzes installed on man-portable air defense systems (MANPADS), targeting small aerial vehicles such as cruise missiles. The method is based on the "kinematic-geometric" relationship between the missile, fragments, and the target, ensuring that the average fragment trajectory passes through the target's center. Approach scenarios are divided into zones, with each zone using a common delay component. The laser beam's inclination angle is aligned with the average fragment trajectory, ensuring delay time independence from miss distance. The approach zones are defined by kinematic relationships, including head-on or tail-chase modes and the azimuth angle. The delay time for each zone is calculated as the average delay across all scenarios within that zone. The method eliminates the effect of miss distance, with delay components dependent only on the missile's direction of motion. A case study applying the model shows minimal error when using the average miss distance, and the results confirm that the fragment stream consistently hits the target across all approach scenarios. The proposed method offers an effective solution for accurately determining delay time and optimizing fuze performance in MANPADS.

Keywords: Small target platform; Initiation delay; Laser fuze; Fuze-warhead coordination; Fixed delay component.

INTRODUCTION

When approaching a target, air defense missiles often do not directly strike the target's surface but instead have a certain miss distance. The fuze emits a laser beam into space and receives reflected signals. It processes these signals and sends a command to activate the warhead. The fragments from the warhead are designed to destroy critical components of the target. Determining the detonation timing to ensure the fragment stream hits the target is the core problem of "fuze-warhead coordination."

To achieve "fuze-warhead coordination," man-portable air defense missiles can be equipped with fixed-delay-time laser fuzes. These fuzes contain several fixed delay components. Depending on the approach scenario, an appropriate delay component is selected for detonation. The delay time depends on the relative velocity and relative position of the missile with respect to the target. Air defense missiles and targets move at very high speeds, resulting in a short interaction time. Therefore, the fuze must process information in real-time. The delay components need to ensure the advantages of fast processing, convenience for the operator, and a design that does not require a complex fuze structure, making them suitable for man-portable air defense systems (MANPADS).

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The time delay components on the fuze cause it to function for a delay time τ after detecting the target. Due to the very high velocity of the missile and the target in a short approach time, the fixed time delay components are suitable for small missiles such as MANPADS. Besides, fixed time delay components have a quick computational advantage and generalized fault tolerance, making them a popular choice for proximity fuzes (Huang *et al.* 2022). However, the methods for calculating fixed time delays are often not studied and publicly available.

This paper presents a calculation method based on the fuze-warhead coordination model to calculate the fixed delay time. The objective of the calculation method is to establish a mathematical relationship between the intersection parameter and the time delay of the laser fuze. This methodology involves using the characteristics of fragmens dispersion to determine the hit location on the target as well. The fixed time delay is a constant value (Hou *et al.* 2022). Nonetheless, variations in intersection parameters typically result in distinct values for the fixed time delay. Thus, it is imperative to investigate the cooperation between proximity fuzes and fragment warheads in the final stages of combat.

The fuze-warhead coordination model is a method that uses the information and relationship between the missile, target, and fragments to detonate the warhead during missile-target combat, maximizing damage to the target (Zhao *et al.* 2017). Some functions of fuze-warhead coordination have been developed. The literature on "the theory of fire efficiency" provided a model for determining the conditions of fuze-warhead coordination considering the target as a mass point (Quy and Hong 2006). When analyzing the fire efficiency of the MANPADS, Elskin *et al.* (2007) mentioned the use of fixed delay components for the proximity fuze but did not present the method for calculating the delay time.

Li *et al.* (2007) developed the features of an explosively formed penetrator (EFP) warhead, established the hit point model, and designed the loitering munitions fuze warhead coordination system. Yang *et al.* (2012) established a fuze-warhead coordination model for ammunition that considers the ammunition's attitude angles as well as the relative motion of the ammunition and the target. Zhao and Chen (2019) generated a fuze-warhead coordination model for early warning aircraft attacks and investigated the impact of common parameters. Wang and Li (2017) investigated the terminal efficiency of three-dimensional fragment-type air-to-air missiles, performing detailed modeling to determine single-shot kill probability, including fuze-warhead calculation. Most of the works above assume that the probability of causing damage to the target is highest at the target-detection point, so they do not use the time delay. The reason is that large weapons have been prioritized; without regard for the shape of the target, almost all targets will be destroyed at the explosion point of the weapon. Historically, the lethality function has often been developed for air-launched weapons, with a huge warhead. Furthermore, Zhang *et al.* (2012) introduced a mathematical model that uses fuze-warhead coordination to calculate the adaptive time delay when an anti-radiation missile targets a radar system. The adaptive time delay method requires time to compute and calculate, so it is only suitable for large missiles, where the final stage of approach is lengthy.

Overall, research on the "fuze-warhead coordination" of laser fuzes mounted on anti-aircraft missiles remains limited. No study has focused on the method of determining fixed delay components in laser fuzes for engaging small-sized targets. This article conducts a comprehensive study and establishes a model to determine the delay components of a laser fuze, ensuring that the warhead fragments hit the target while minimizing the number of delay components. By analyzing the kinematic-geometric relationship between the anti-aircraft missile and the small-sized target, a formula for calculating the delay time for a specific engagement scenario is developed.

By considering all engagement scenarios, a model is established to identify the delay components of the fuze. Before launching the missile, the operator (shooter) cannot determine the miss distance; therefore, a solution is needed to eliminate the effect of the missile's relative position to the target on the delay time. The engagement scenarios are divided solely based on kinematic relationships: head-on or tail-chase engagement modes and azimuth angles. The delay time of a delay component is the average delay time at characteristic points within the engagement zone. The model is applied to solve a problem for a hypothetical small-sized target and evaluate the method.

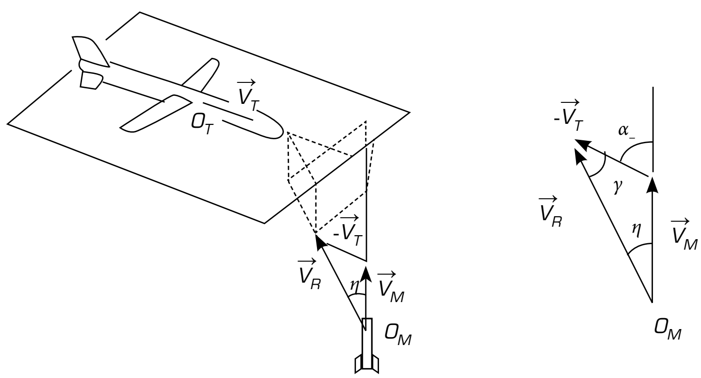
The paper is organized as follows: the "Kinematic-geometric relationship" section presents the relative motion between the missile and the target, the characteristics of small targets, and the classification of approach scenarios. The "Fuze-warhead coordination" section outlines the conditions for fragments to pass through the center of the target and derives the delay time formula. The "Determination of delay components" section proposes a model to identify the fixed delay components of the fuze, applies it to a specific problem, and evaluates the error. Finally, the last section provides conclusions and remarks.

The "kinematic-geometric" relationship

The missile, fragments, and target are moving objects in space. Each missile-target approach scenario is characterized by the relative position and velocity of the missile in relation to the target, which is determined by the crossing parameters. The kinematic-geometric relationship represents the connection between these crossing parameters as the missile approaches and destroys the target.

Kinematic relationship

The kinematic relationship refers to the relationship between the velocity of the missile, the velocity of the fragments, and the velocity of the target. This relationship is considered in the context of the relative motion with respect to the target. When the missile engages the target, the kinematic relationship between the missile and the target is illustrated in Fig. 1.



Source: Elaborated by the authors.

Figure 1. The relative velocity of the missile to target.

The relative velocity of the missile relative to the target is defined as:

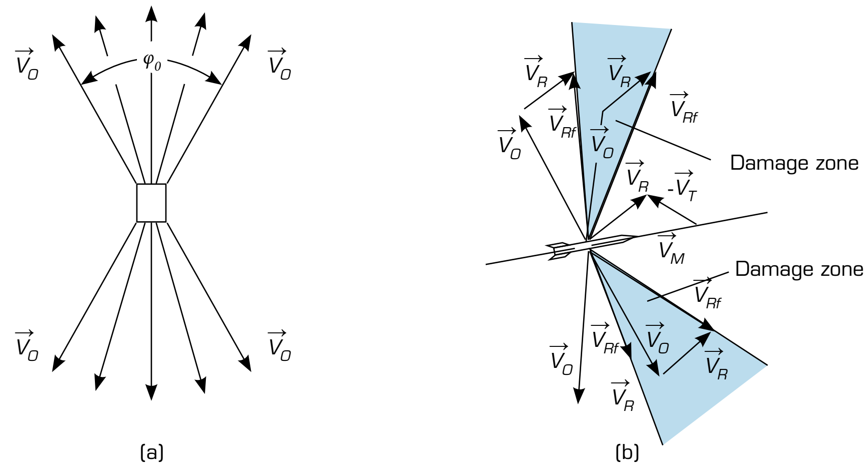
$$\vec{V}_R = \vec{V}_M - \vec{V}_T \tag{1}$$

The azimuth angle (η) is the angle between the relative movement direction of the target to the missile. Alternatively, the approach angle (α) represents the relative movement direction of the missile to the target.

Relative motion of fragments with respect to the target

When a warhead detonates while the missile is stationary, the fragments are ejected in all directions, creating a "fragment dispersion zone," also known as a "static explosion." This dispersion zone is geometrically represented by two cones whose apexes coincide at the explosive's center. These cones define the boundaries of the fragment spread. However, when the missile is in flight and detonates near the target, the situation changes. In this case, because the detonation occurs very close to the target, the effects of air resistance on the fragments can be ignored due to the short distance. The total velocity of the fragments becomes a combination of two components: static explosion velocity and missile velocity. Thus, the relative velocity of the fragments concerning the target is the sum of the fragment velocity from the static explosion and the relative velocity of the missile as it approaches the target, as illustrated in Fig. 2.

$$\vec{V}_{Rf} = \vec{V}_0 + \vec{V}_R \tag{2}$$



Source: Elaborated by the authors.

Figure 2. The diagram of the warhead fragment. (a) static dispersion zone; (b) the relative fragment velocity.

The relative velocity vectors of the fragments define the damage zone. This is the area where the fragments are likely to impact following the detonation of the warhead. At the moment of detonation, if the target is located within this damage zone, the fragments will strike the target, causing damage. The shape and size of the damage zone depend on the relative velocities and the detonation geometry, ensuring that even without a direct hit, the fragments can still effectively reach the target.

The bisector of this damage zone, which represents the midpoint of the spread of the fragments, is referred to as the "average fragment path." This path indicates the direction in which most of the fragments will travel, and it serves as a critical reference for determining whether the missile's proximity fuze detonation will result in a successful strike on the target. Ensuring the target is within the average fragment path at the moment of detonation increases the probability of successfully hitting and neutralizing the target.

Geometric relationship

The geometric relationship represents the correlation of the positions of the missile, the laser beam, and the target during the missile's approach. This relationship encompasses the spatial positioning of the fragments produced upon detonation. The geometric relationship is expressed in a relative coordinate system (Fig. 3). This relative coordinate system is an orthogonal system attached to the target, with its origin at the target's center O_T. The X-axis is parallel to the missile's relative velocity vector. The Y-axis lies in the vertical plane and is perpendicular to the X-axis, while the Z-axis forms a right-handed coordinate system with the X and Y axes. The plane O_TYZ, called the "image plane," is used to evaluate the missile's "miss distance."

The "image plane O_TYZ" is crucial for visualizing and calculating the missile's "miss distance R" as it approaches the target. As the missile approaches the target, the missile's center O_M moves along a trajectory parallel to the X-axis. The missile's trajectory intersects this plane at the "arrival point O", which is critical for determining engagement success. Due to random errors, the arrival point lies within a dispersion ellipse on the image plane O_TYZ. The miss distance R is the distance between the missile's arrival point O and the target's center O_T. The miss distance R and the angle ξ determine the position of the arrival point on the image plane.

As the missile approaches the target, the proximity fuze emits laser beams into the environment. The detection angle Ω is the angle between the laser beam and the missile's vertical axis. MANPADS with laser proximity fuzes typically have a fixed detection angle Ω (Fig. 4). The missile rotates around its vertical axis, causing the laser beam to sweep and form a cone-shaped observation region. The location of this observation cone is determined by the missile's position in the relative coordinate system.

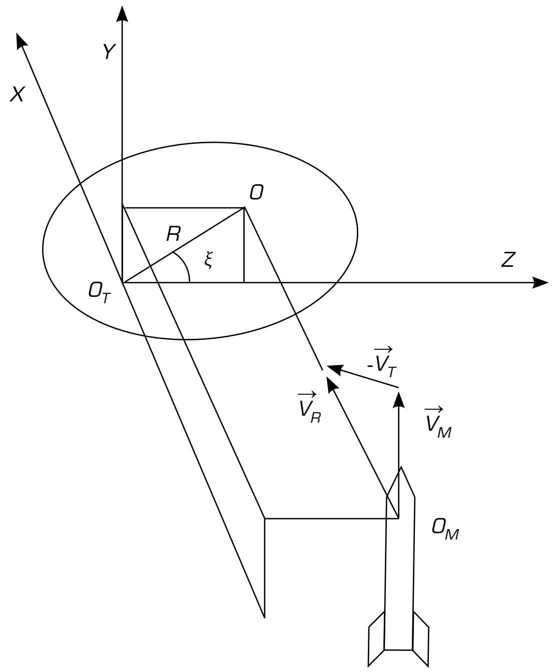


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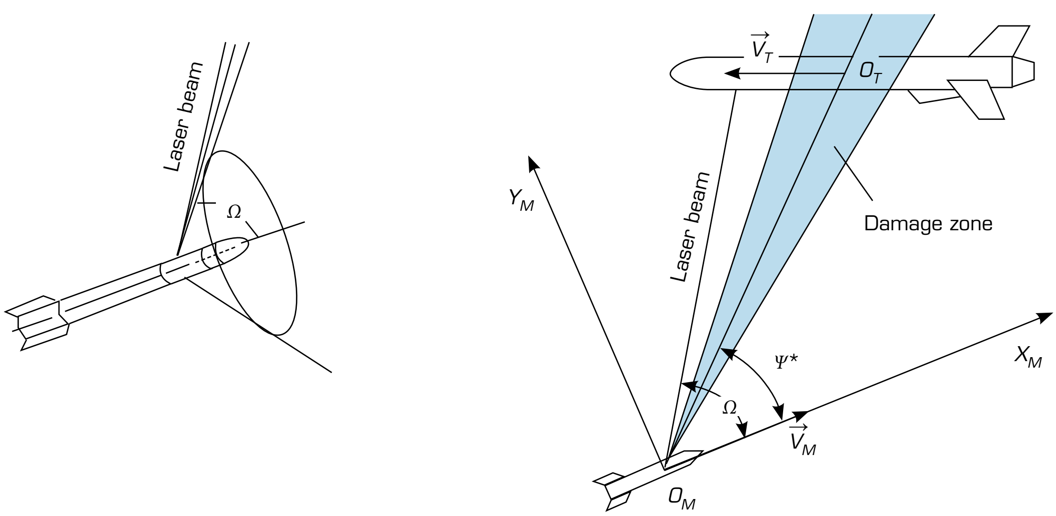
Figure 3. Relative coordinate system.



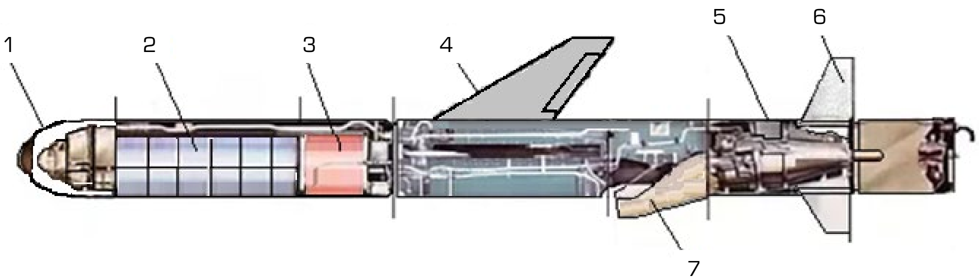
Source: Elaborated by the authors.

Figure 4. Diagram of the fuze detection field.

Since the arrival point depends on guidance errors, the miss distance R and angle ξ are random variables that the operator cannot predict. When determining the delay component, a solution must be found to eliminate the influence of the miss distance R and angle ξ.

Types of targets

MANPADS are designed to engage various types of targets, including fighter jets, helicopters, unmanned aerial vehicles (UAVs), and cruise missiles. These targets differ in shape, size, flight characteristics, and the arrangement of internal components. By comparing the characteristics of these targets, they can be categorized into two groups: large targets (this group includes fighter jets, helicopters, and large UAVs) and small targets (this group consists of cruise missiles and small UAVs; for small targets, cruise missiles can be selected as a representative example). The structure of a cruise missile is illustrated in Fig. 5.



Source: Elaborated by the authors.

Figure 5. Diagram of the layout of the parts of the cruise missile. 1: guidance computer; 2: warhead; 3: fuel compartment; 4: wings; 5: turbo jet engine; 6: stable tail; 7: air intake.

Structure of the cruise missile

The structure of a cruise missile has the following characteristics:

Critical compartments: all compartments of the missile body contain critical components, while the wings of the cruise missile are not considered vital components in the context of MANPADS approach. Therefore, when calculating the effectiveness of fire, the missile can be modeled as a long cylindrical object.

Constant speed: the speed of the target remains nearly constant and can be considered a constant value V_T = const.

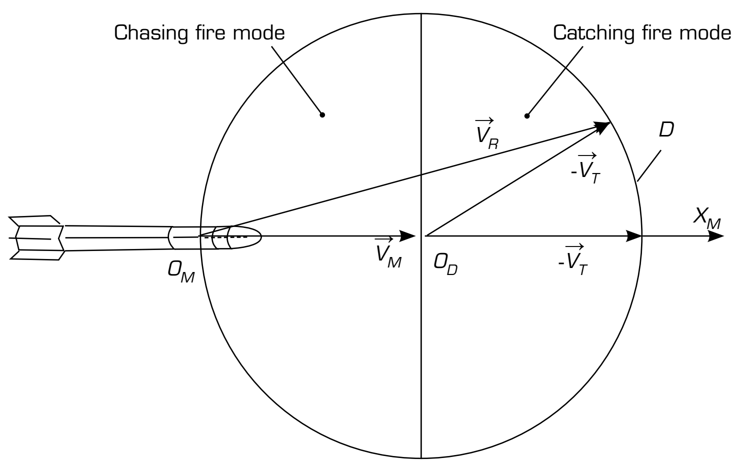
Center of gravity positioning: the center of the target is positioned horizontally at the root of the lifting wings. The distance from the nose of the target to the center is denoted as l_1, and the distance from the tail of the target to the center is denoted as l_s.

Missile relative velocity zone

The velocity of both the surface-to-air missile and the target is subject to specific limitations in magnitude, meaning that their maximum and minimum speeds are constrained. As a result, the relative velocity vector of the missile, which is the relative velocity vector of both the missile's and the target's velocities, is only able to change within a specific zone (zone D).

This restricted zone D of the missile's relative velocity vector represents the possible approach scenarios, based on the kinematic relationship between the missile and the target. Within this region, the missile can adjust its trajectory and speed relative to the target to optimize its catching ion path. The shape and size of this region are determined by the limits on both the missile's and the target's velocities, as well as the angles at which the missile approaches the target.

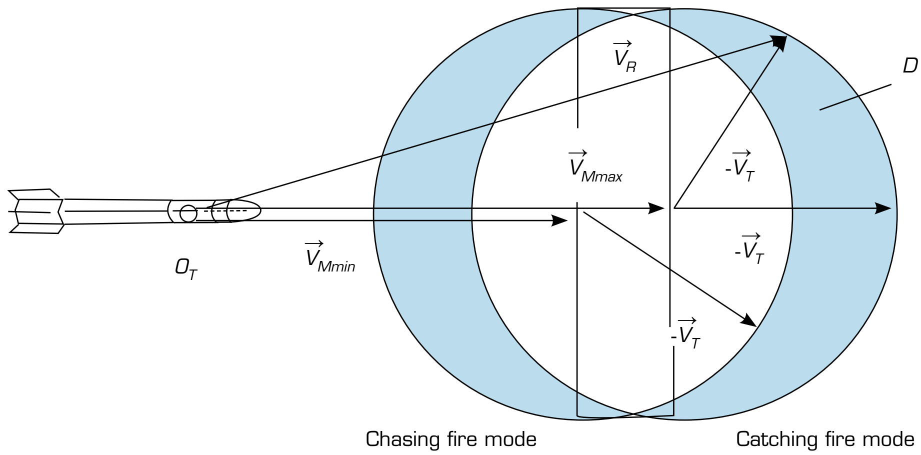
Under the condition that the vector $V_T \leq V_M$, the restricted zone D is represented as a circular area (Fig. 6).



Source: Elaborated by the authors.

Figure 6. Missile's relative velocity restricted zone D when $V_T \leq V_M$.

In the case of the surface-to-air missile approaching a small target, the missile's velocity varies within the specified range, denoted as V_{Mmin} to V_{Mmax}. The velocity of the cruise missile is considered constant: V_M = const. The restricted zone D contains the apex of the missile's relative velocity vector $\overrightarrow{V_R}$, as illustrated in Fig. 7.



Source: Elaborated by the authors.

Figure 7. Relative velocity vector in all approach cases for small target $\overrightarrow{V_R}$.

Fire modes

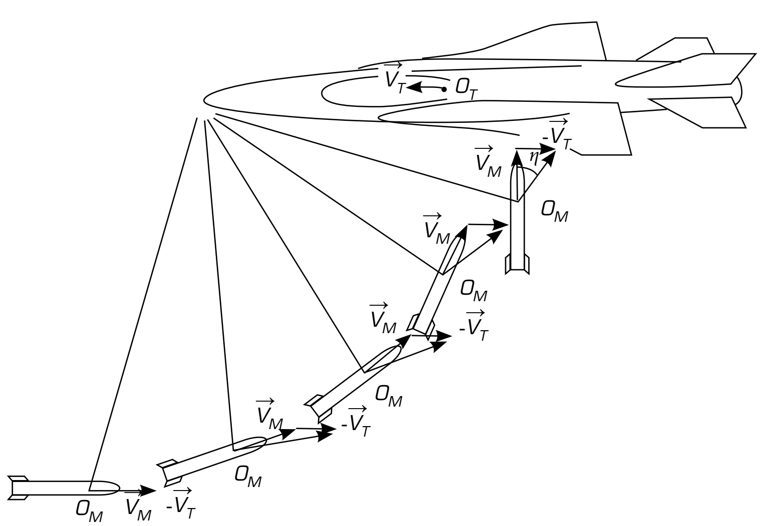
Based on the angle between the missile's velocity vector and the target's velocity vector, two fire modes are identified: "chasing mode" and "catching mode." These modes are also illustrated within the region D:

Chasing mode: the missile approaches from behind the target. The left half of the restricted zone D corresponds to the chasing mode.

Catching mode: the missile approaches from in front of the target. The right half of the restricted zone D corresponds to cases of the catching mode.

Azimuth angle

The azimuth angle represents the relative motion direction of the target compared to the missile. In all approach scenarios, the azimuth angle η varies within a specific range. Figure 8 illustrates the variation of the azimuth angle η as the missile approaches the target in the catching mode.



Source: Elaborated by the authors.

Figure 8. Intersection cases of different azimuth angles η in catching fire mode.

Segmentation of approach areas

Each approach scenario requires a specific delay time to ensure the missile detonates at the optimal moment for target destruction. Addressing every scenario individually would require the fuze to contain a large number of fixed delay components, significantly complicating its design and increasing the complexity of the system.

To simplify the structure of the fuze and minimize the number of fixed delay components, approach scenarios are grouped into approach areas. In each area, the scenarios share similar delay times, allowing them to be managed by a single fixed delay component. This approach reduces the number of necessary elements while maintaining effective targeting.

The division of these approach areas is based on the kinematic and geometric relationships between the missile and the target. Factors such as the missile's velocity, the target's velocity, and the relative approach angles are considered to ensure that the delay times within each area remain close enough to use the same fixed delay component. This method optimizes the missile's effectiveness while simplifying the overall design of the fuze.

Geometric relationships

The relative position of the missile to the target is influenced by the miss distance, which is random and difficult to predict. Therefore, a solution must eliminate the effect of these positional variations on the delay time. When dividing the approach areas, the relative positions between the missile and the target are intentionally disregarded, simplifying the calculation and use of delay components in the fuze system. This allows for consistent performance across various scenarios without being impacted by the specific location of the missile relative to the target at detonation.

Kinematic relationships

The approach scenarios are divided according to the fire mode and the azimuth angle.

According to the azimuth angle, the scenarios are categorized into areas of large azimuth angles and areas of small azimuth angles. The boundaries of these areas are based on the parameters of the maximum azimuth angle and the limiting azimuth angle.

The maximum azimuth angle η_{max} is achieved when the missile flies at its minimum speed $V_{M\min}$ and the target flies at its maximum speed $V_{T\max} = V_T$. It can be calculated as:

$$\eta_{max} = \arctan\left(\frac{V_{M\max}}{V_{T\min}}\right) \tag{3}$$

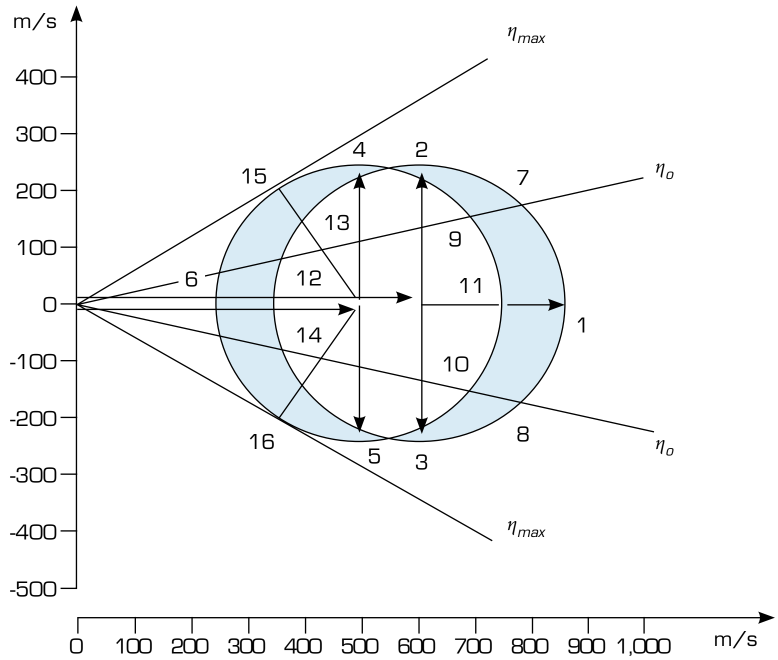
The limited azimuth is the boundary between two areas, which can be selected:

$$\eta_0 = \frac{\eta_{max}}{2} \tag{4}$$

Approach area division

The area D is symmetric about the horizontal axis, meaning that symmetrical approach scenarios across this axis will have equal delay times. Therefore, it suffices to consider the approach scenarios in the upper half of area D.

Based on the fire mode and azimuth angle, four distinct approach areas can be defined, as illustrated in Fig. 9. These areas correspond to different combinations of fire modes (chasing and catching) and azimuth angles (large and small). Each area will allow the use of a common fixed delay component, optimizing the design and functionality of the fuze while ensuring effective target approach.



Source: Elaborated by the authors.

Figure 9. Typical characteristic points and approach areas on round *D*.

An overview of missile approach modes is presented in Table 1. These divisions allow for more straightforward calculations and implementations of fixed delay components in the fuze design while accounting for the different dynamics of each approach scenario.

Table 1. Overview of missile approach modes.

Area	Fire mode	Azimuth angle	Explanation
Upper left	Chase	Large azimuth	Missile approaches from behind the target at a significant angle
Upper right	Catching	Large azimuth	Missile approaches the target from the front at a large angle
Lower left	Chase	Small azimuth	Missile approaches from behind at a smaller angle
Lower right	Catching	Small azimuth	Missile approaches the target from the front at a smaller angle

Source: Elaborated by the authors.

Characteristic points

The points located on the boundary of the approach areas possess the maximum or minimum intersection parameters within that area. These points are regarded as the characteristic points of the approach zones.

Characteristic points for the approach areas can be selected based on their positions on the limit of the region defined in area *D*. The significance of these characteristic points lies in their ability to represent critical conditions for the missile-target approach.

The average of the delay time at the characteristic points can be taken as the required delay time of the approach area. This average delay time provides a representative value that can be employed to set the fuze timing for the proximity fuze in the missile. By using this average, the system can effectively engage targets across the designated approach area while maintaining operational efficiency.

Coordination of fuze and warhead

Model assumptions

The formula for calculating the time delay is established based on the "kinematic-geometric relationship" between the missile, the fragment, and the target. The missile and target are moving objects in space. Establishing the equations for the system involving the missile, laser beam, target, and fragment in three-dimensional space is highly complex.

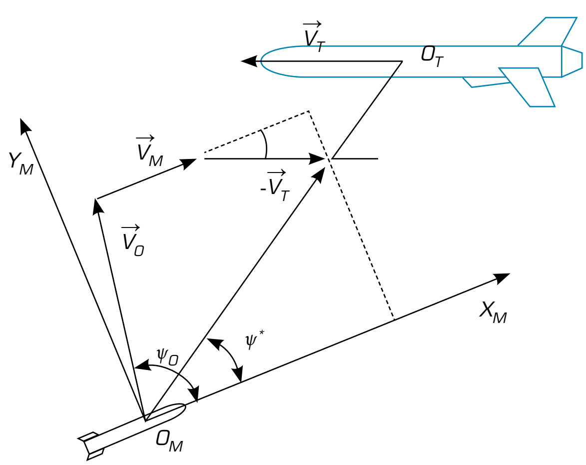
When calculating the delay time, the following assumptions are used:

- The target moves in a uniform straight line at velocity V_T, while the missile moves in a uniform straight line at velocity V_M.
- The velocity vector of the missile aligns with its longitudinal axis, and the velocity vector of the target aligns with its longitudinal axis (neglecting the angle of attack).
- The distance from the detonation point to the target's surface is very small, allowing air resistance to be neglected and assuming the relative trajectory of the fragment is a straight line.
- The laser light is emitted with a very narrow divergence angle, approximated as a beam projected into space.
- The target is modeled as a cylindrical shape, with the laser beam projected and scanned along the cylinder's axis.
- The projections of the velocity vectors, missile trajectory, laser beam, and fragment distribution onto the "encounter" plane are used to simplify the "kinematic-geometric relationships" in three-dimensional space into two-dimensional space.

Effective coordination between the fuze and the warhead is critical for maximizing the lethality and effectiveness of missile systems. This coordination involves several factors, including the timing of detonation and the warhead's performance characteristics. To derive the formula for calculating the delay time, it is first necessary to determine the conditions for fragments to pass through the center of the target.

Conditions for fragments to pass through the center of the target

Given the intersection parameters and the initial velocity of the fragment at the moment of the "static" explosion, it is necessary to determine the conditions under which the fragment will pass through the center of the target, denoted as O_T. When the fragment strikes the center of the target, the kinematic relationship between the missile, the fragments, and the target is illustrated in Fig. 10.



Source: Elaborated by the authors.

Figure 10. The fragment hits the center of the target.

The relative velocity of the fragments relative to the target is determined by the equation:

$$\vec{V}_{Rf} = \vec{V}_0 + \vec{V}_M - \vec{V}_T \tag{5}$$

Projecting vectors onto axes X_M and axes Y_M, we have

$$\tan\psi^* = \frac{V_o \sin\psi + V_T \sin\alpha_T}{V_o \cos\psi + V_T \cos\alpha_T + V_M} \tag{6}$$

Projecting a vector onto the axis and coordinate axis, we have:

$$\tan\psi^*_{mM} = \frac{V_{mt}.\sin\psi_m - v_M.\sin\alpha_T}{V_{mt}.\cos\psi_{mt} + V_M.\cos\alpha_T + V_T} \tag{7}$$

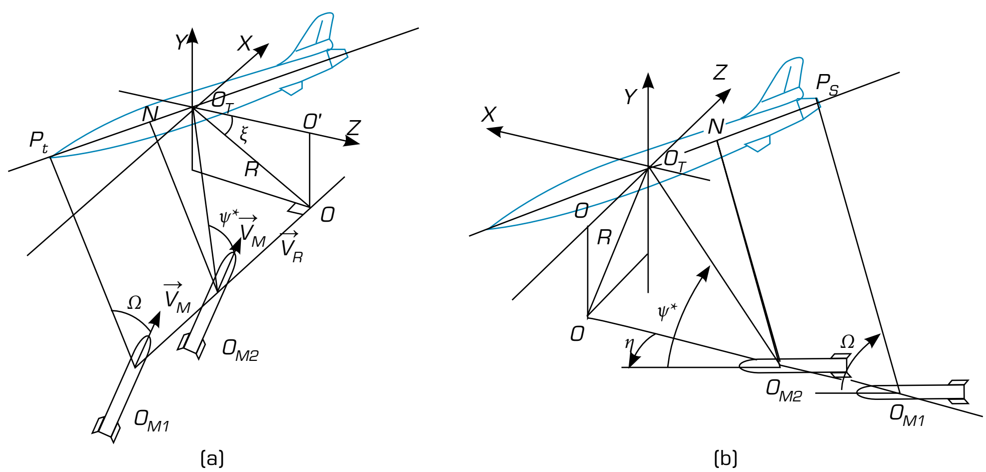
where Ψ^* is the angle of relative velocity of the projectile passing through the center of the target,

V_0 is the velocity of the fragment when detonating is static, and α_T is the angle of approach of the missile to the target.

Formula for calculating time delay

The formula for calculating the time delay is established based on the kinematic-geometric relationship between the missile, the fragment, and the target. In the terminal approach trajectory, due to tracking errors, the missile approaches the target with a miss distance R. The fuze emits a laser beam into the environment and receives a return signal at point O_{M1}.

In the catching fire mode, the laser beam is projected from the tip P_t of the target and scans toward the center O_T of the target (Fig. 11a). In the "chase" fire mode, the laser beam is projected from the tail P_s of the target and scans toward the center O_T of the target (Fig. 11b). After a time delay τ, the fuze will detonate the warhead at point O_{M2}. The resulting fragment will then strike the target.



Source: Elaborated by the authors.

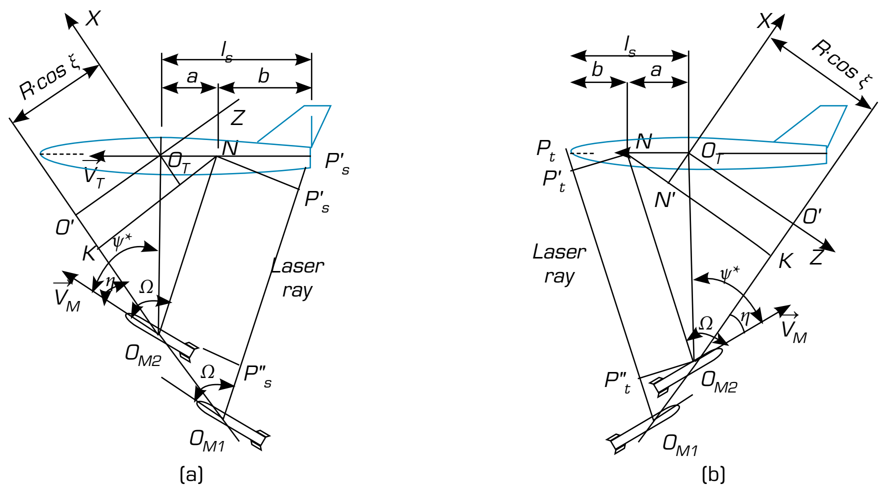
Figure 11. Kinematic-geometric relationships of the missile, fragments, and target. (a) catching fire mode; (b) chasing fire mod.

Criteria for selecting the detonation point

The fuze must detonate at a moment that creates the highest damage effect on the target. The fragment dispersion area of the shoulder-fired anti-air missile is represented as a cone with a small opening angle (ϕ_0 = 14'). The direction of the damage area can be determined by the direction of the average fragment trajectory. The critical components of the target are usually located near its center. The criterion for selecting the detonation moment is defined as the fuze detonating at a point where the average fragment trajectory passes through the center of the target.

Projection on the encounter plane ("meeting" plane)

The missile, target, and fragments are all moving objects in space, and the kinematic-geometric relationships are quite complex. By projecting the trajectory and velocity of the missile, the laser beam, and the average fragment distribution onto the encounter plane, the kinematic-geometric relationship of the missile, laser beam, and fragment in relative motion with respect to the target is obtained, as shown in Fig. 12.



Source: Elaborated by the authors.

Figure 12. Projection of velocity vector, trajectory, laser beam, fragments onto the plane O_TXZ. (a) chasing fire mode; (b) catching fire mode.

Formulating the delay time

The delay time τ is calculated from the moment the fuze receives the return signal until the moment it detonates.

Along the longitudinal axis of the target, the distance b represents the length of the laser sweep along the target body up to the detonation point; this distance bb depends on the delay time τ. The distance aa is the relative displacement of the fragment in the direction of the target's longitudinal axis to the center of the target O_T.

Under the given intersection conditions, the delay time τ of the fuze can be calculated through the kinematic-geometric relationship.

Delay time in the chase mode

Distance a: consider the projection of the vertical axis of the target to the relative trajectory of the missile $O_{M1}O_{M2}$.

$$O_{M2}K = KN\cot(Q) - (R\cos\xi + a\sin(\alpha_T - \eta))\cot(\alpha - \eta),$$
$$O_{M2}O' = R\cos(\xi(\psi^* - \eta)),$$
$$KO' = a\cos(\alpha - \eta)$$

where α is the approach angle.

Distance a is defined as a condition: the *OK* projection of distance a is equal to the projection of the $O_{M2}O'$ piece of the projectile minus the projection of the laser $O_{M2}K$.

From the conditions: $KO' = O_{M2}O' - O_{M2}K$.

Hence:

$$a = R\cos\xi \cdot \frac{\cot(\psi^* - \eta) - \cot(\Omega - \eta)}{\sin(\alpha - \eta)\cot(\Omega - \eta) + \cos(\alpha - \eta)} \tag{8}$$

Distance b: during the delay time τ, the missile moves in an orbit relative to the velocity V_R.

$$O_{M1}\,O_{M2} = V_R \cdot \tau.$$

where τ is the delay time.

Projection of the $O_{M1}O_{M2}$ missile moving down perpendicular to the laser,

$$P_S''.O_{M2} = V_R.\tau.\sin(\Omega - \eta)$$

Projection distance b down perpendicular to the laser,

$$P_S^{'}N = b, \sin\left(\alpha - \Omega\right)$$

The projection of the missile's displacement along the relative trajectory in the laser beam's scanning path b,

$$P_S^{''}O_{TN2} = P_S^{'}N$$

$$V_R \cdot \tau \cdot \sin\left(\Omega - \eta\right) = b \cdot \sin\left(\alpha - \Omega\right)$$

Hence:

$$b = \frac{V_R \cdot \tau \cdot \sin\left(\Omega - \eta\right)}{\sin\left(\alpha - \Omega\right)} \tag{9}$$

Calculate the delay time τ: Delay time τ is measured from the moment the laser beam first begins to illuminate on the target surface to the moment the detonator activates the warhead. At the detonation point, the laser beam shines at point N. On the vertical axis of the target $l_s = a + b$, the position a and b in l_s is expressed to calculate the delay time:

$$\tau = \frac{1}{V_R}\left[l_s - R.\cos\xi.\frac{\cot\left(\psi - \eta\right) - \cot\left(\Omega - \eta\right)}{\sin\left(\alpha - \eta\right).\cot\left(\Omega - \eta\right) + \cos\left(\alpha - \eta\right)}\right].\frac{\sin\left(\alpha - \Omega\right)}{\sin\left(\Omega - \eta\right)} \tag{10}$$

Delay time in the catching mode

The calculation of the delay time in pick-up catching mode is similar to that in chase catching mode. The main difference is that the laser scans from the target nose to the center of the target.

Distance a:

$$a = R.\cos\xi.\frac{\cot\left(\psi - \eta\right) - \cot\left(\Omega - \mu\right)}{\sin\left(\alpha - \eta\right).\cot\left(\Omega - \eta\right) + \cos\left(\alpha - \eta\right)} \tag{11}$$

Distance b:

$$b = \frac{V_R.\tau.\sin\left(\Omega - \eta\right)}{\sin\left(\alpha + \Omega\right)} \tag{12}$$

On the target vertical axis $l_t = a + b$, the delay time is:

$$\tau = \frac{1}{V_R}\left[l_t - R.\cos\xi.\frac{\cot\left(\psi + \eta\right) - \cot\left(\Omega + \eta\right)}{\sin\left(\alpha - \eta\right).\cot\left(\Omega + \eta\right) + \cos\left(\alpha - \eta\right)}\right].\frac{\sin\left(\alpha + \Omega\right)}{\sin\left(\Omega + \eta\right)} \tag{13}$$

Equations 8 and 9 show that if the angle of detection Ω is equal to the angle of inclination of the fragments, then the delay time Ψ* time τ will not depend on the degree of deflection R.

Determining delay time elements

Model for determining fuze delay time elements

Problem statement

Given a type of MANPADS equipped with a laser fuze, the task is to determine the delay time elements of the fuze that ensure the fragment hits the target in all approach scenarios while minimizing the number of delay time elements.

Input parameters

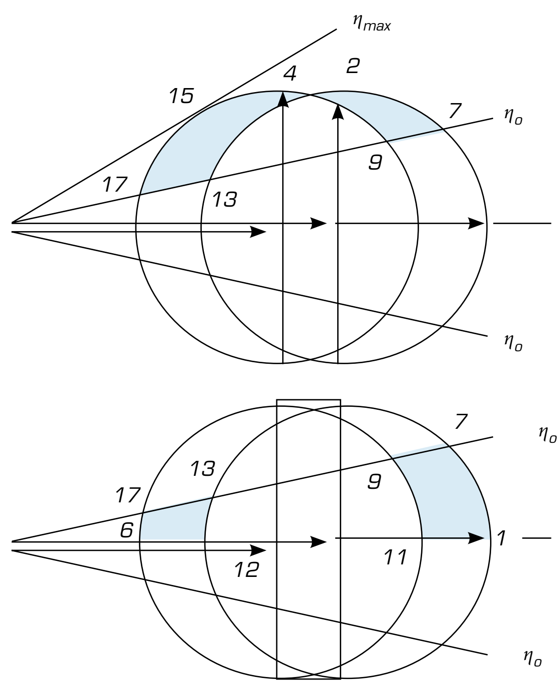
The input parameters for this problem include the characteristics of the missile and the target:

- Maximum missile velocity V_{Mmax} and minimum missile velocity V_{Mmin}
- Open angle of the static fragment area ϕ_0
- Velocity of the fragment when in a static state V_o
- Damage radius of the warhead r
- Maximum target velocity V_{Tmax} and minimum target velocity V_{Tmin}
- Distance from the nose of the target to its center l_t
- Distance from the tail of the target to its center l_s

Procedure for determining delay time elements

The determination of the delay time elements for the fuze is carried out in the following steps:

- Construct the limit area D: using the velocity parameters of the missile and the target, create the limit area D. The small target is assumed to have a constant velocity.
- Select detection angle Ω: choose the detection angle Ω of the fuze, calculate the maximum azimuth angle η_{max}, and select the limiting azimuth angle η_0 (the boundary between the large and small azimuth angle areas).
- Divide approach cases into approach areas: segment the approach scenarios into distinct approach areas and select characteristic points located on the boundaries of these areas. For small targets, this segmentation is illustrated in the corresponding Fig. 13.



Source: Elaborated by the authors.

Figure 13. Time delay at characteristic points.

- Calculate the delay time at characteristic points of the approach areas.
- Calculate the necessary delay time for each approach area, determining the necessary delay time for the approach areas by averaging the delay times at the characteristic points.
- The necessary delay time for each approach zone is the average delay time of the approach cases belonging to that zone. Additionally, characteristic points located on the boundary between two adjacent approach zones will be used to calculate the delay time for both zones.

- The duration for the delay components is the necessary delay time of the approach zones; it can be adjusted as needed.

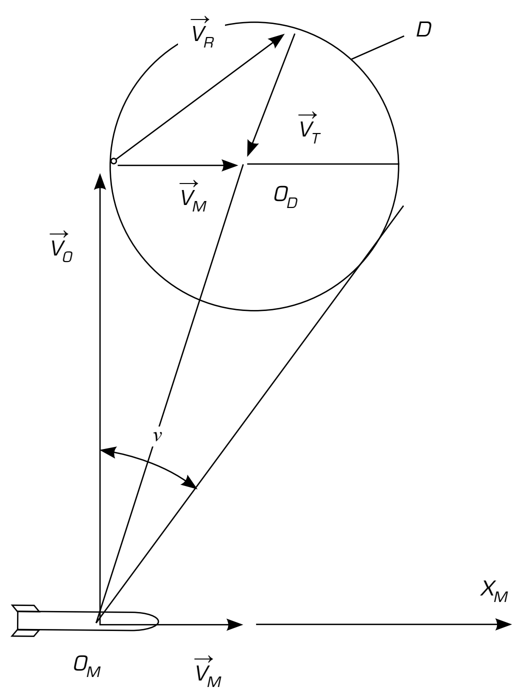
Selection of the detection angle for the fuze

The relative position of the missile with respect to the target is a random variable, influenced by the missile's guidance error, making it very difficult to predict. When designing the fuze, it is essential to eliminate the relative position of the missile concerning the target (geometric relationship) from the selection of the delay time elements. A structural solution for the fuze can be employed to choose the detection angle Ω in such a way that variations in the miss distance R and angle ξ do not affect the delay time.

According to Eqs. 8 and 9, if the detection angle Ω is equal to the inclination angle of the fragment Ψ^*, then the delay time τ will not depend on the miss distance R and angle ξ. For different approach scenarios, the angle Ψ^* will vary within the range $\Psi^* = 61° – 87°$. A fuze with a constant detection angle Ω cannot ensure the condition $\Omega = \Psi^*$ in all approach scenarios.

The average distribution line of fragments in all approach scenarios is a straight line that starts from the center of the fuze O_M and passes through the center O_D of the area *D* (the boundary of the relative velocity vector area of the missile). The line $O_M O_D$ has an inclination angle Ψ_D (Fig. 14). If the detection angle of the fuze is chosen as $\Omega = \Psi_D$, then the angle Ψ^* for the various approach scenarios will not significantly differ from the detection angle Ω.

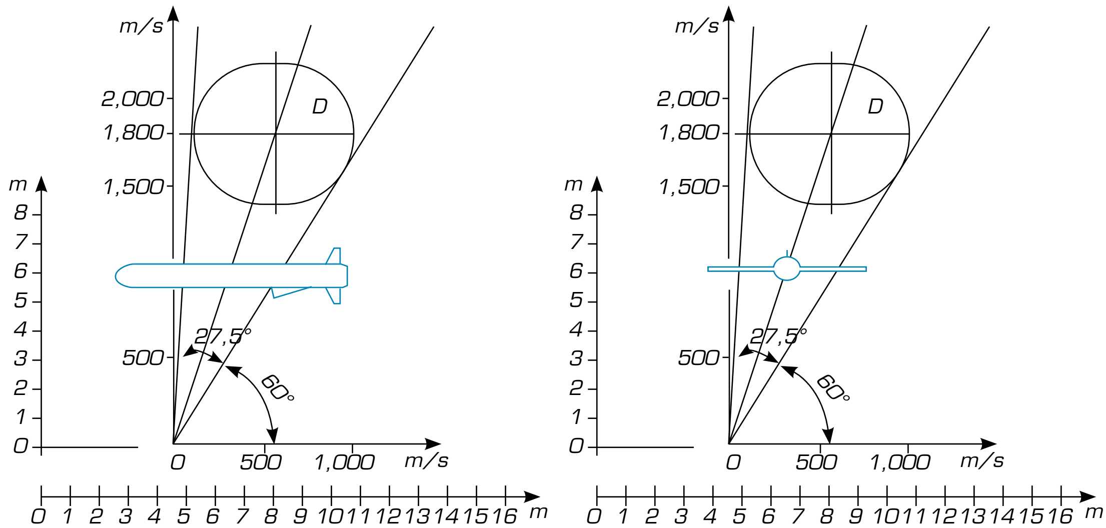
Figure 14 shows the fragment distribution midline for all approach cases. The fragment distribution lines all approach the center O_T of the target. The miss distance *R* and angle ξ will not significantly affect the delay time, allowing the average miss distance R_{avr} to be used for calculating the delay time τ.



Source: Elaborated by the authors.

Figure 14. The fragment distribution midline of all approach cases.

It is possible to check the deviation from the target center of the average fragment distribution line for various approach scenarios. Place the target diagram on the velocity diagram such that the center of the target lies on the average fragment distribution line $O_M O_D$ of all approach scenarios, with the miss distance being R_{max} (Fig. 15). Draw two rays that encompass the area *D*, indicating the limits of the fragment distribution lines for the approach scenarios.



Source: Elaborated by the authors.

Figure 15. Fragments dispersion to the hypothetical target.

Application problem

The model for determining the delay time elements of the laser fuze is applied to surface-to-air missiles and hypothetical targets.

Parameters of the missile and target

The input parameters of the missile and target used for calculations are provided in Table 2.

Table 2. Missile and target parameters.

Parameter	Value
Missile	
Maximum missile speed (V_{Mmax})	600 m/s
Minimum missile speed (V_{Mmin})	500 m/s
Fragment effective killing radius (r)	5.5 m
Target	
Maximum target speed (V_{Tmax})	400 m/s
Minimum target speed (V_{Tmin})	0 m/s
Fragment parameters in static detonation	
Static fragment dispersion angle (Φo)	14°
Fragment initial velocity (V_0)	1,800 m/s

Source: Elaborated by the authors.

The miss distance R is allowed to vary within the effective killing radius of the warhead:

$R_{max} = r = 5.5m$. The average miss distance R_{avr} is chosen as 2.3 m.

Calculate the delay time at the characteristic points and the necessary delay time for the approach areas. The calculation results are presented in Table 3.

Table 3. Results of time delay on area D for small size target (V_T = const).

Zone	Point	Fire mode	VT	α_T	Time
1	6	Chasing	V_{Tmin}	π	0.0168
	12	Catching	V_{Tmax}	0	0.0095
	13	Chasing (small azimuth)	V_{Tmin}	η_{gh}	0.0013
2	15	Catching	V_{Tmin}	$\alpha_T > 0$	0.0055
	17	Chasing	V_{Tmin}	η_{gh}	0.0151
	13	Chasing (large azimuth)	V_{Tmin}	η_{gh}	0.0027
3	4	Catching	V_{Tmin}	$\pi/2$	0.0017
	2	Catching	V_{Tmax}	$\pi/2$	0.0029
	7	Catching	V_{Tmax}	η_{gh}	0.0037
4	7	Catching	V_{Tmax}	η_{gh}	0.0029
	9	Catching	V_{Tmax}	η_{gh}	0.0037
	1	Catching	V_{Tmax}	0	0.0038
	11	Small azimuth	$V_M = 0$	-	0.0045

Source: Elaborated by the authors.

Evaluation of the error of the method

In the calculations, the average of the delay times at the characteristic points was taken as the required delay time for the engagement zones. However, this approach introduces an error, as the fragment group may deviate from the target. To evaluate the effectiveness of fragment impacts on the target, the following steps can be taken:

Overlay the target diagram: superimpose the target diagram onto the velocity plot, ensuring that the center of the target is aligned with the average fragment distribution line for all approach scenarios. The miss distance is defined as R_{max}.

Define the fragment distribution region: draw two bounding rays to enclose the region D, representing the limits of the fragment distribution across various approach scenarios.

The inspection is carried out with the maximum miss distance R_{max} = 5.5 m.

Impact of using the average deflection

When calculating the delay time at characteristic points, the average deflection R_{avr} was used. To evaluate the error caused by using the average deflection, the delay time is calculated with the maximum deflection R_{max} and compared with the delay time calculated using the average deflection R_{avr}. The difference between the two times obtained is the error that needs to be determined. The results of the calculations are presented in Table 4.

Table 4 shows that the average error resulting from using the mean deviation R_{avr} is around 10%. At characteristic points 7 and 13, the maximum error reaches up to 16%. However, when determining the necessary delay time for the zones, the average value will be taken, resulting in an error that is smaller than the maximum error, but it is still essential to verify that the fragment flow strikes the target.

Table 4. Delay time evaluation for approach zones.

Approach zone	Point	Delay time (R_{avr} = 2.33 m)	Delay time (R_{max}= 5.5 m)	Deviation	Rate (%)
1: Chasing, small azimuth	6	0.0168	0.0200	0.0032	16.0
	12	0.0095	0.0086	0.0009	10.5
	13	0.0013	0.0015	0.0002	16.0
2: Chasing, large azimuth	15	0.0055	0.0059	0.0004	6.7
	17	0.0151	0.0139	0.0012	8.6
	13	0.0013	0.0015	0.0002	16.0
	4	0.0027	0.0028	0.0001	3.5
3: Catching, large azimuth	2	0.0017	0.0015	0.0002	13.3
	7	0.0029	0.0025	0.0004	16.0
	9	0.0037	0.0034	0.0003	8.8
4: Catching, small azimuth	7	0.0029	0.0025	0.0004	16.0
	9	0.0037	0.0034	0.0003	8.8
	1	0.0038	0.0033	0.0005	15.2
	11	0.0045	0.0040	0.0005	12.5

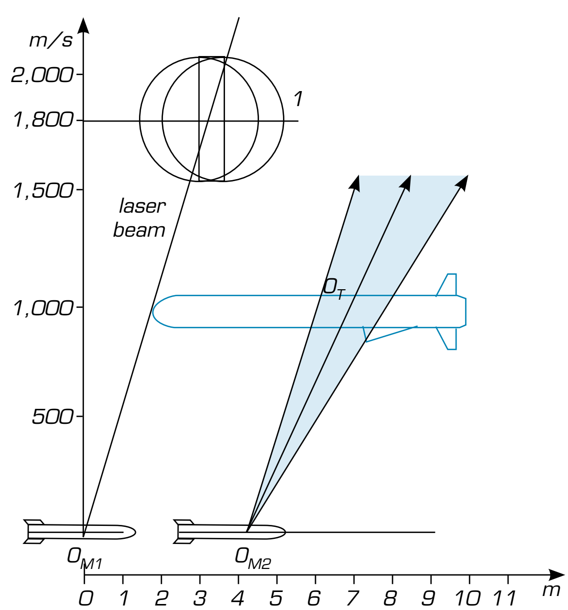
Source: Elaborated by the authors.

Impact of average delay time

When determining the delay time elements for the approach zones, the delay time for each zone is the average of the delay times at the characteristic points. Using the average value can create discrepancies, potentially causing the fragment flow to deviate from the target. It is necessary to check the condition: at the characteristic points, the fragment flow must strike the target. This verification is conducted with the maximum deviation $R_{max} = r = 5.5$m. The results of the verification are as follows:

Case for characteristic point 1

Characteristic point 1 corresponds to the "catching" catching mode and an azimuth angle η=*0*. Since the delay time of the component exceeds the required delay time, the average fragment distribution shifts backward from the center O_T of the target. As shown in Fig. 16, the fragment flow enters the vulnerable area of the target.

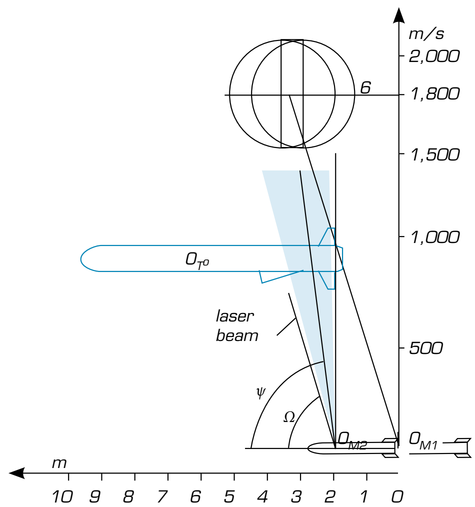


Source: Elaborated by the authors.

Figure 16. Fragment group in the case of point 1.

Case for characteristic point 6

Characteristic point 6 corresponds to the “chasing” catching mode and an azimuth angle $\eta=0$. The delay time of the delay time component is less than the required value, causing the fragment flow to shift behind the center O_T of the target, as illustrated in Fig. 17. This area also represents a vulnerable region of the target.

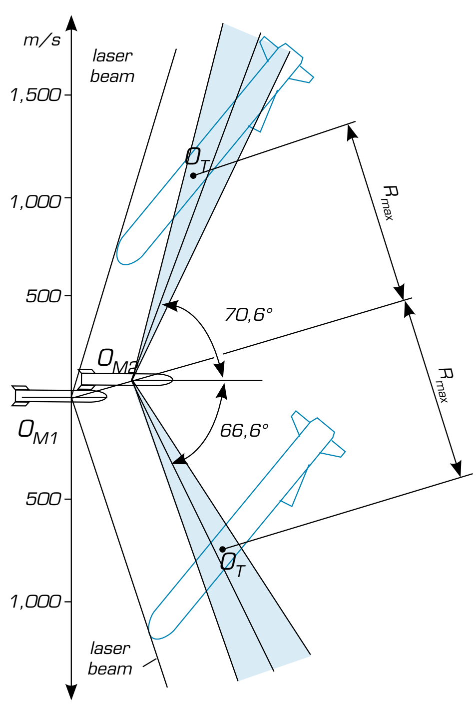


Source: Elaborated by the authors.

Figure 17. Fragment group in the case of point 6.

Case for characteristic point 7

Characteristic point 7 corresponds to the “catching” catching mode and an azimuth angle $\eta=\eta_0$. There are two target positions, as shown in Fig. 18. For both target positions, the fragment flow successfully strikes the target, and the average fragment distribution line closely approaches the center O_T of the target.



Source: Elaborated by the authors.

Figure 18. Fragment group in the case of point 7.

In general, the fragment flow successfully strikes the target in all approach scenarios. The experimental results demonstrate that using the average value of the delay time at characteristic points, as the necessary delay time for the approach area is appropriate.

CONCLUSION

The article establishes a model to determine the fixed delay components of a laser fuze mounted on a man-portable air defense missile. By analyzing the kinematic-geometric relationship between the missile, the warhead fragments, and the small-sized target, it defines the restricted region of the missile's relative velocity. From this, a method is proposed to classify engagement scenarios kinematically based on head-on or tail-chase modes and azimuth angles.

From the kinematic-geometric relationship, the fuze-warhead coordination problem is solved, ensuring that the fragments pass through the target's center. By projecting the velocity vectors, missile trajectory, laser beam, and fragments onto the encounter plane, the kinematic-geometric relationships in three-dimensional space are transformed into relationships in the plane. Hence, a formula for calculating the delay time is established for both head-on and tail-chase engagement scenarios.

For the determination of delay components in the fuze, it is necessary to eliminate the influence of the miss distance from the delay time. By choosing the detection angle of the laser fuze Ω to be equal to the angle of the fragment dispersion midline of all approach cases Ψ_D, the delay time will not depend on the miss distance R. The approach scenarios are divided into four different engagement zones. The average value of the delay times at the characteristic points (located at the boundary of the region) is taken as the delay time for the approach area.

The model for determining the fixed delay components is proposed to ensure that the fragment flow hits the target in all engagement scenarios while minimizing the number of delay components. The model is applied to solve a problem for a hypothetical small-sized target. The evaluation of the method demonstrates that the proposed model is valid and appropriate. Through case study, it is shown that a laser fuze with fixed delay components can be mounted on a man-portable air defense missile, where the interaction time is minimal, necessitating quick decision-making without complexity.

The model is built for a small target with a constant flight speed. If the target flies at different speeds, the above method can still be applied, but the shape of the region D must be adjusted accordingly.

CONFLIT OF INTEREST

Nothing to declare.

AUTHORS' CONTRIBUTION

Conceptualization: Nguyen HL; **Methodology:** Nguyen HL; **Validation:** Trung DP and Truong SN; **Writing - Original Draft:** Nguyen HL; **Writing - Review & Editing:** Nguyen HL and Van HT; **Final approval:** Nguyen HL.

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All data sets were generated or analyzed in the current study.

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