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Reliability Importance Analysis of a Multi-State System with Binary Components Using Survival Signature

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ABSTRACT

Reliability assessment of a multi-state system with binary state components (MSS-BC) is highly practical because the assumption that the system has a binary state system (BSS) is often unrealistic in many engineering applications. The main research problem is to determine the state of the MSS-BC based on the minimal path required for system operation and to evaluate its components' importance. Such information is essential for purposes such as component prioritization, reliability improvement, and risk reduction (RR), allowing for the identification of a system's weaknesses or critical components and the quantification of the impact of their failures on an MSS-BC. In this paper, a new reliability assessment approach for MSS-BC is presented, based on disjoint product forms of minimal path sets and survival signature. It also introduces methods for the Birnbaum importance (BI), improvement potential (IP), and RR measures using these concepts. Both the numerical case and the case study presented a driving subsystem in aerospace engineering to demonstrate the applicability of the approach for MSS-BC. The proposed technique shows clear superiority and potential for applications in aerospace engineering.

Keywords: Structural reliability; Multi-state system with binary state components; Sum of disjoint products method; Survival signature; Component importance; Risk reduction.

INTRODUCTION

The reliability of multi-state systems (MSSs) is crucial in aerospace applications due to their complexity. Unlike traditional binary states systems (BSSs), which can either be functional or failed, MSSs can operate at various performance levels, reflecting real-world conditions. Aerospace systems experience different operational states, such as cruise, ascent, and descent. Evaluating reliability across these states provides valuable insights into system performance. By modeling these variations, engineers can assess risks and make informed design and operational decisions, ensuring safety and compliance with regulations. Additionally, analyzing MSS reliability helps identify critical components necessary for maintaining high performance, as well as those that can tolerate degradation without leading to failures (Kuo and Zhu 2012; Lisnianski *et al.* 2010; Natvig 2010; Qin *et al.* 2016).

The primary research problem is to determine the status of the MSS-binary state components (BC), focusing on the minimal path required for effective system operation. Gertsbakh and Shpungin (2020) provide definitions related to networks, describe different types of network failures, and offer an overview of various criteria for assessing network failure. A network is considered to be in a state of perfect functioning if the maximum flow from the source to the sink meets or exceeds a specific given value. Conversely, the network is in a state of complete failure if it does not meet this threshold. The disjoint product forms divide the

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system's reliability into multi-states, which are the states of the system during its lifetime. By associating the disjoint product forms with the survival signature, a more accurate study can be made to calculate the reliability of MSS-BC based on the survival signature. Many researchers have studied algorithms for constructing discrete product sums (Abraham 1979; Datta and Goyal 2017; Jane and Yuan 2001; Mutar 2023; 2025).

The signature of a system is essential for comparing the structures of coherent systems. Kochar *et al.* (1999) examined the necessary conditions for one system's lifetime to be greater than another system's lifetime in terms of stochastic ordering, likelihood ratio ordering, and hazard rate ordering. Contemporary advances in the use of the system signature concept have been reported in Samaniego (2007). However, this approach's limitation is that it assumes all system components are of the same type. Since real systems usually consist of different component types, analyzing such systems using the system signature becomes challenging. Coolen and Coolen-Maturi (2012) introduced the survival signature concept as an improvement over the system signature. Unlike the system signature, the survival signature does not rely on the restriction to one component type. This means that the characteristics of the components no longer need to be independent and identically distributed (i.i.d.) types. When only one component type exists, the survival signature is closely associated with the system signature. Gertsbakh and Shpungin (2011) proposed creating a strong link between the cumulative D-spectra and the derived number of system failure sets, emphasizing the potential insights this connection could provide. Eryilmaz and Tuncel (2016) expertly presented the concept of survival signature, effectively tailored for a particular class of unrepairable homogeneous MSSs. This innovative approach offers valuable insights into system reliability and performance. Marichal *et al.* (2017) analyzed the combined signature of MSSs consisting of binary-state components. Mi *et al.* (2020) presented an importance analysis based on survival signature used to analyze the reliability of a dual-axis pointing mechanism for communication satellites, which is a commonly used satellite antenna control mechanism. Ge and Zhang (2020) identified the essential components of a complex system using a survival signature. The feasibility of the proposed approach is demonstrated through an actual production system. Yi *et al.* (2021) discussed the theoretical aspect of system signatures for multi-state coherent or mixed systems with i.i.d. binary-state components. Yi *et al.* (2022a) presented the joint signatures of BSSs and MSS-BCs. Additionally, several examples are provided to illustrate and verify the theoretical results established. Yi *et al.* (2022b) considered coherent MSSs that can be viewed as a combination of series, parallel, or recurrent connections of multi-state modules with either binary or multi-state components. Qin and Coolen (2022) proposed a reliability evaluation of MSSs, computing methods of survival signature are studied for reliability analysis of several different systems. Yang *et al.* (2024a) proposes a survival signature-based reliability framework for an imprecise MSS. Yang *et al.* (2024b) developed a survival signature-based reliability framework for an MSS, taking into account both dependence and uncertainty. Chang *et al.* (2023) introduced a generalized reliability technique for complex systems that uses survival signatures and stochastic processes to model degradation, allowing for reliability analysis without failure data. Chang *et al.* (2024) presented a generalized reliability model specified using structural analysis techniques and the survival signature, enabling the proposed method to be applied to different structural systems.

Understanding the importance index measures for components is crucial for assessing the necessary components within a system and identifying the most critical ones. Various techniques exist for conducting importance analysis, with the primary aim of determining the influence of one or multiple components on the system's reliability (Armstrong 1997; Birnbaum 1968; Kuo and Zhu 2012; Mehni and Mehni 2023; Zaitseva and Levashenko 2013; Zheng *et al.* 2023). In these studies, the importance index measures are computed based on a structure function (Armstrong 1997; Birnbaum 1968; Kuo and Zhu 2012; Zaitseva and Levashenko 2013), Markov model (Kuo and Zhu 2012; Mehni and Mehni 2023; Zheng *et al.* 2023), universal generation function (Zhou *et al.* 2019), and Monte-Carlo simulation (Vaisman and Sun 2021). Several studies delve into the importance of index measures calculated based on the survival signature and the techniques and algorithms for calculating various importance index measures based on the system's representation through a survival signature (Di Maio *et al.* 2023; Huang *et al.* 2019; Mi *et al.* 2020; Mutar 2024; Mutar and Hassan 2025; Rusnak *et al.* 2022; 2024). Mi *et al.* (2020) investigates common cause failures. Huang *et al.* (2019) discuss the computation of BI based on the survival signature. The definition of the system critical state is studied in Di Maio *et al.* (2023) and Rusnak *et al.* (2022). Rusnak *et al.* (2024) propose a technique for calculating structural importance measures in BSS utilizing survival signatures and direct partial logical derivatives (DPLD).

Additionally, Mutar and Hassan (2025) use an approach to calculate structural importance measures in MSS-BC employing the survival signatures and DPLD.

The determination of the system state is regarded as a max-flow problem (Gertsbakh and Shpungin 2011; 2020). Gertsbakh and Shpungin (2011) investigated a multidimensional analog of the D-spectrum specifically defined for binary coherent systems. In the research, disjoint product forms of minimal path sets were utilized to determine the system's state based on the operating minimal path sets. Qin and Coolen (2022) defined system states based on the number of components in the minimal path sets. In contrast, this paper defines system states using operating minimal path sets, which provides a more accurate representation. Yang *et al.* (2024b) also categorized system states by the number of components in the minimal path sets and utilized the survival signature. However, this paper offers a broader range of system states by employing operating minimal path sets and presenting an updated version of the survival signature. Feng *et al.* (2016) established the importance measures using the survival signature for binary systems. In this paper, the finding on importance measures using the survival signature were extended to MSS-BC.

The novelty of this study lies in developing a new reliability assessment approach for MSS-BC, based on disjoint product forms of minimal path sets and survival signature. It also introduces methods for reliability importance analysis, BI, IP, and RR measures using these concepts. Unlike traditional reliability analysis methods, this approach evaluates the state of the MSS with BCs based on the minimal path required for system operation. It evaluates its components' importance based on survival signature. This paper expands on the definition of the MSS-BC, introduced by Qin and Coolen (2022), by incorporating concepts based on disjoint product forms of minimal path sets calculations referenced in sources (Mutar 2023; 2025). Therefore, this method defines an MSS-BC model based on the disjoint product forms of minimal path sets to form states with the number of minimal paths required for system operation. In other words, rather than defining system states based on the number of components in each minimal path in Qin and Coolen (2022), this study focuses on the unique states of each minimal path by utilizing disjoint product forms (providing more detailed and accurate states of a system based on the number of minimal path sets). The research presents a novel way to calculate the BI, IP, and RR measures for MSS-BC by analyzing survival signatures.

The remainder of this paper is organized as follows: first, a state characterization of MSS-BC based on disjoint product forms of minimal path sets and a methodology for reliability analysis based on survival signatures. Next, several component importance index measures, including the BI, IP, and RR measures for the MSS-BC model using survival signatures, are presented. Consequently, a numerical example is included to illustrate the proposed techniques in detail. Additionally, a real-world application of the MSS-BC model in aerospace engineering is discussed. Finally, the concluding remarks are provided.

MSS WITH BINARY-STATE COMPONENTS

Generating state of negation component

Consider a system consist of n components $c_1, c_2, \dots, c_n$ and m minimal path sets $\mathcal{MP}_1, \mathcal{MP}_2, \dots, \mathcal{MP}_m$. The Boolean variables can be defined by arithmetic operations using disjunction form of components as follows:

$$\vee_{i=1}^{n} c_i = \overline{c_1} + c_1\overline{c_2} + \cdots + c_1 \dots\ c_{n-1}\overline{c_n} \tag{1}$$

where $\overline{c_i} = 1 - c_i$ is negation of i-component. Also, the disjunction form can be defined on all minimal path sets. Based on Eq. 1, the complement set $\mathcal{MP}_z - \mathcal{MP}_j = \{c_i \in \mathcal{MP}_z \text{ and } c_i \notin \mathcal{MP}_j, 1 \le z < j \le m\}$ of the minimal path sets can be define as:

$$\mathcal{MP}_z - \mathcal{MP}_j = \vee_{i=1}^{n} c_i \tag{2}$$

Then, the disjoint product form set $\mathcal{D}_j$ of the j - th minimal path sets, based on Eq. 2, can be expressed as:

$$\mathcal{D}_j = \prod_{z=1}^{j-1} \left(\mathcal{MP}_z - \mathcal{MP}_j\right) \mathcal{MP}_j \tag{3}$$

Therefore, to generate the state of the negation component, assume that $\overline{c_i} = (1 - c_i)^{s_{i_j}}$ and, based on Eq. 3 for all $\overline{c_i} \in D_j$, the state of the negation component can be defined as follows:

$$s_{i_j} = \begin{cases} 1, & if\ \overline{c_i} \in \mathcal{D}_j \\ 0, & otherwise \end{cases} \tag{4}$$

Finally, the structure function $\phi(\boldsymbol{c})$ of the system, according to Eq. 3, can be represented as:

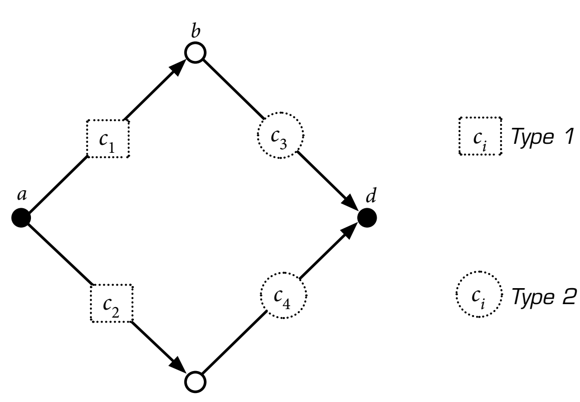
$$\phi(\boldsymbol{c}) = \sum_{j=1}^{m} \mathcal{D}_j \tag{5}$$

The techniques used in Eqs. 1–5 to convert this representation into a sum of disjoint simple products (Abraham 1979).

Definition 1: let $\mathcal{D} = \{\mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_j, \dots, \mathcal{D}_m\}$ be the set of distinct product forms of all minimal path sets while the system is operational. Assume that $D_0 = 1 - \sum_{j=1}^{m} \mathcal{D}_j$ is "complete failure" state of the considered system. The state of the system is determined by the random variable $J \in \{0,1, \dots, j, \dots, m\}$. A value of 1 corresponds to the optimal state, j represents any intermediate condition, and 0 indicates a state of complete failure.

For instance, consider the parallel-series system consisting of four binary-state components $c = (c_1, c_2, c_3, c_4)$, as depicted in Fig. 1. Especially, components c_1 and c_2 belong to type 1, while components c_3 and c_4 belong to type 2. The system has two operational minimal path sets: $\mathcal{MP}_1 = \{c_1, c_3\}$ and $\mathcal{MP}_2 = \{c_2, c_4\}$. Consequently, using Eq. 3 and 4, it is obtained that $\mathcal{D}_1 = c_1 c_3$ and $\mathcal{D}_1 = \overline{c_1} c_2 c_4 + c_1 c_2 \overline{c_3} c_4$, as depicted in Fig. 2.

According to Definition 1, the system has three distinct states, represented by $J \in \{0, 1, 2\}$. Specifically, a complete failure corresponds to $J = 0$, perfect functionality corresponds to $J = 1$, and the intermediate state is represented by $J = 2$. Assuming



Source: Elaborated by the authors.

Figure 1. A parallel-series MSS-BC.

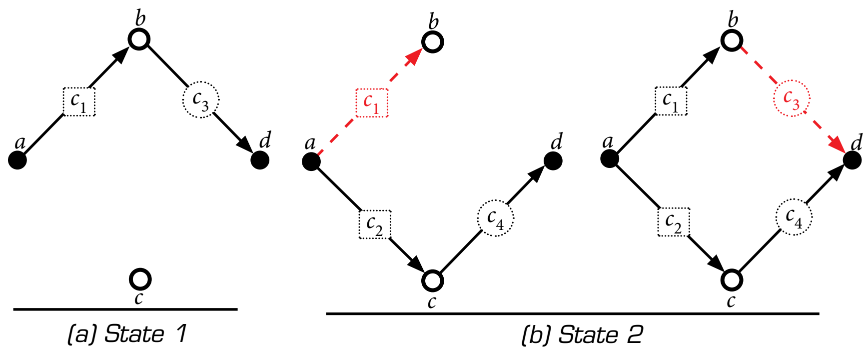


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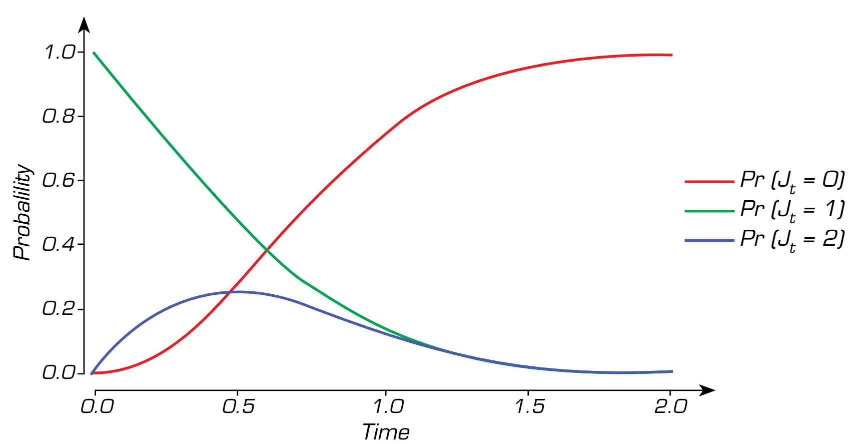
Figure 2. The operation states of the parallel-series MSS-BC are described using disjoint product forms. The red dashed line represents the state of the negation component, whereas (a) represents the first operational state of the system and (b) represents the second operational state of the system.

that the failure distribution for type 1 components follows an exponential distribution with an expected value of 1, we obtain the reliability functions $R_1(t) = e^{-t}$ and $R_2(t) = e^{-t}$. For type 2 components, a Weibull distribution with a shape parameter of 2 and a scale parameter of 1 is assumed, resulting in the reliability functions $R_3(t) = e^{-t^2}$ and $R_4(t) = e^{-t^2}$. According to Definition 1, the probabilities of the system existing in various states can be effectively plotted, as shown in Fig. 3.

Figure 3 shows how the states of the system are represented using the disjoint product forms of minimal path sets. In the next subsection, an algorithm will be developed to find the disjoint product forms of minimal path sets.



Source: Elaborated by the authors.

Figure 3. The probabilities of the parallel-series MSS-BC for different states at time t.

Disjoint product form

The more useful and efficient techniques involve defining a structure function as a sum of non-elementary disjoint products. The number of these products is small (Abraham 1979).This approach can be achieved by executing simple algebraic processes on the initial form derived from the set of all minimal path sets (Datta and Goyal 2017; Mutar 2023). The disjoint product form algorithm can be given as follows:

Step 0: input all minimal path sets $\mathcal{MP}_1, \mathcal{MP}_2, \dots, \mathcal{MP}_m$ of the system with order collection.

Step 1: for $\mathcal{MP}_1$ *assume that* $v_{1,1} = \{c_i = 1 | c_i \in \mathcal{MP}_1\}$ *and* $s_{i_1} = 0$.

Step 2: define $\mathcal{D}_{j,0} = \mathcal{MP}_j$ where $1 \leq z < j \leq m$ as follows:

1. If $\mathcal{MP}_z - \mathcal{MP}_j = \emptyset$, then $\mathcal{D}_{j,0} = \{\mathcal{MP}_j\}$, so $v_{j,1} = \{c_i = 1 | c_i \in \mathcal{MP}_j\}$ *and* $s_{i_1} = 0$.
2. If $c_i \in \mathcal{MP}_z$ *and* $\overline{c_i} \in \mathcal{MP}_j$ go to step 2.1.
3. If $\mathcal{MP}_z - \mathcal{MP}_j = \bigvee_{i=1}^{n} c_i$ then $\mathcal{D}_{j,z} = \bigvee_{i=1}^{n} c_i\, \mathcal{MP}_j$, so $v_{j,z} = \{c_i = 1 | c_i \in (\mathcal{MP}_z - \mathcal{MP}_j)\mathcal{MP}_j\}$ and $s_{i_j} = 1$.

Step 3: repeat step 3 to form $\mathcal{D}_{j,z}$ for $j = 2, \ldots, m$.

In the above algorithm, the vectors $v_{j,z}$ where $j, z = 1,2, \ldots, m$ representing the state $j \in \{1,2, \ldots, m\}$ of the system are extracted. The state s_{i_j} of the negated components ($\overline{c_i}$) where $i = 1,2, \ldots, n$ and $j = 1,2, \ldots, m$ is also extracted as in Eq. 4. To effectively illustrate how the algorithm works, consider Fig. 1. The following steps are carefully outlined:

Step 0: the input consists of all minimal path sets $\mathcal{MP}$ as follows: $\mathcal{MP}_1 = \{c_1, c_3\}$ and $\mathcal{MP}_2 = \{c_2, c_4\}$.

Step 1: for $\mathcal{MP}_1 = \{c_1, c_2\}$: it is obtained that $v_{1,1} = (1,0,1,0)$ and $s_{i_1} = 0$, $k = 1,2,3,4$.

Step 2: for : $\mathcal{MP}_2 = \{c_2, c_4\}$

let $\mathcal{D}_{2,0} = \{c_2, c_4\}$

Step 2.3: $\mathcal{MP}_1 - \mathcal{D}_{2,0} = \{c_1, c_3\}$, *then* $\mathcal{D}_{2,1} = \{\bar{c}_1, c_2, c_4\}$*and* $\mathcal{D}_{2,2} = \{c_1, c_2, \bar{c}_3, c_4\}$ then $v_{2,1} = (0,1,0,1)$ and $v_{2,2} = (1,1,0,1)$ and $s_{1_1} = s_{3_2} = 1$.

The algorithm's outputs for the system in Fig. 1 are vectors representing the operating system states, as illustrated in Fig. 2. In the next subsection, the survival signature will be used to study the states of the system in detail based on the outputs of the proposed algorithm.

Survival signature and disjoint product form

A system with two states and various components K ≥ 2 types is crucial for optimal performance. The system consists of n components, where n_k represents the number of -types of components, satisfying $k \in \{1, 2, \ldots, K\}$ and $\sum_{k=1}^{K} n_k = n$. Components are i.i.d. of the same type, and the random times of components of different types are entirely independent. The state vector $c = (c^1, c^2, \ldots, c^k, \ldots, c^K)$ can group its components of the same type together, with the sub-vector $c^k = (c_1^k, c_2^k, \ldots, c_{n_k}^k)$ describing the states of components of type *k*. The survival signature of a system that performs with exactly l_k of its components of type *k* is represented as $\Phi(l_1, l_2, \ldots, l_k, \ldots, l_K)$, where $l_k \in \{0,1, \ldots, n_k\}$. Consider $\binom{n_k}{l_k}$ state vectors c^k, each with precisely l_k out of its n_K components $c_i^k = 1$. These state vectors are represented by S_l^k for components of type *k*. The collection of all state vectors for the entire system, where $l_k = \sum_{i=1}^{n_k} c_i$, is represented by $S_{l_1, l_2, \ldots, l_k, \ldots, l_K}$. Then:

$$\Phi(l_1, l_2, \ldots, l_k, \ldots, l_K) = \left[\prod_{k=1}^{K} \binom{n_k}{l_k}\right]^{-1} \times \sum_{\boldsymbol{c} \in S_{l_1, \ldots, l_k}} \phi(\boldsymbol{c}) \tag{6}$$

Let $c_t^k \in \{0, 1, \ldots, n_k\}$ denote the number of components of type *k* in the system that function at time *t* > 0. Based on Eq. 4 for all $\overline{c_i} \in D_j$, the state of the negation components of type *k* can be defined as follows:

$$s_{k_j} = \begin{cases} 1, \mathit{if}\ \overline{c_k} \in D_j \\ 0, \mathit{otherwise} \end{cases} \tag{7}$$

Definition 2: let $H = (H_1, H_2, \ldots, H_k, \ldots, H_K)$ represent the operational state vector of the components, where H_k denotes the operational status of the components of type k, $H_k = (h_{1_k}, h_{2_k}, \ldots, h_{i_k}, \ldots h_{n_k})$, i_k means the ith component of type k. If the i_k-th component functions, then and $h_{i_k} = 1$ and $h_{i_k} = 0$, if it does not function. The survival signature of the system is:

$$\Phi(l_1, l_2, \ldots, l_k, \ldots, l_K) = \left[\prod_{k=1}^{K} \binom{n_k}{l_k}\right]^{-1} \times \sum_{\boldsymbol{H} \in S_{l_1, \ldots, l_k}} \phi(\boldsymbol{H}) \tag{8}$$

The computation of the survival signature for a parallel-series system with two types of components is shown in Fig. 1, and it can be calculated using Eq. 6. In Fig. 1, there are $(n_1 + 1) \times (n_2 + 1) = 3 \times 3 = 9$ combinations of l_1 and l_2, as detailed in Table 1. An example of calculating the vector H at states $j = 1,2$ based on the proposed algorithm and Eqs. 7 and 8 are also provided, with results in Table 2.

Table 1. Survival signatures of parallel-series system.

l_1	l_2	$\Phi(l_1, l_2)$
0	[0,1,2]	0
[1,2]	0	0
1	1	1/2
1	2	1
2	[1,2]	1

Source: Elaborated by the authors.

Table 2. Survival signatures of parallel-series MSS-BC at different states.

State (j)	H	l_1	l_2	$\Phi(l_1, l_2)$	s_{1_j}	s_{2_j}
1	(1,0,1,0)	1	1	1/4	0	0
2	(0,1,0,1)	1	2	1/4	1	0
	(1,1,0,1)	2	1	1	0	1

Source: Elaborated by the authors.

Assuming independent failure times for components of various types and i.i.d. failure times for components of the same type with a given reliability function $R_k(t)$ for components of type k, the reliability function can be deduced based on Eq. 7 for $l_k \in \{0, 1, \ldots, n_k\}$ and $s_{k_j} \in \{0,1\}$ where $k = 1, 2, \ldots, K$ and $j = 1,2, \ldots, m$:

$$Pr\left(\cap_{k=1,2,\ldots,K}(c_t^k = l_k)\right) = \prod_{k=1}^{K} Pr(c_t^k = l_k) = \prod_{k=1}^{K}\left(\binom{n_k}{l_k}(R_k(t))^{l_k}(1 - R_k(t))^{s_{k_j}}\right) \tag{9}$$

Suppose that T denotes the system's random failure times. Consequently, the probability that the system will be operational at t > 0 based on Eqs. 6 and 9 is as follows:

$$Pr(T > t) = \sum_{l_1=0}^{n_1} \ldots \sum_{l_K=0}^{n_K} \Phi(l_1, l_2, \ldots, l_k, \ldots, l_K)\, Pr\left(\bigcap_{k=1,2,\ldots,K}(c_t^k = l_k)\right)$$

$$= \sum_{l_1=0}^{n_1} \ldots \sum_{l_K=0}^{n_K}\left(\Phi(l_1, l_2, \ldots, l_k, \ldots, l_K)\prod_{k=1}^{K} Pr(c_t^k = l_k)\right)$$

$$= \sum_{l_1=0}^{n_1} \ldots \sum_{l_K=0}^{n_K}\left(\Phi(l_1, l_2, \ldots, l_k, \ldots, l_K)\prod_{k=1}^{K}\left(\binom{n_k}{l_k}(R_k(t))^{l_k}(1 - R_k(t))^{s_{k_j}}\right)\right) \tag{10}$$

The key benefit of Eq. 10 is that it fully separates the information about the system's structure from the information about the failure times of its components. Moreover, integrating the distribution of failure times is simplified by the assumed independence of failure times for different component types. To accurately assess a system's reliability, it is essential to obtain the survival signature through an analysis of the system's structure (Mutar 2022; Samaniego 2007).

To determine the survival function of the MSS-BC at a certain state $J = j$, it is necessary to first obtain the survival signature for the entire system at that particular state. The survival signature of the MSS-BC is the likelihood that the system maintains a state $J = j$, assuming l_k components of type k are present. This probability is represented by the symbol $\Phi_{J=j}(l_1, l_2, \ldots, l_k, \ldots, l_K)$. Given the system state J_t at time $t > 0$, the probability of the system being in state j can be defined as follows:

$$Pr(J_t = j) = \sum_{l_1=0}^{n_1} \ldots \sum_{l_K=0}^{n_K} \Phi_{J=j}(l_1, l_2, \ldots, l_k, \ldots, l_K)\, Pr\left(\bigcap_{k=1,2,\ldots,K} (c_t^k = l_k)\right)$$

$$= \sum_{l_1=0}^{n_1} \ldots \sum_{l_K=0}^{n_K} \left(\Phi_{J=j}(l_1, l_2, \ldots, l_k, \ldots, l_K) \prod_{k=1}^{K} Pr(c_t^k = l_k)\right)$$

$$= \sum_{l_1=0}^{n_1} \ldots \sum_{l_K=0}^{n_K} \left(\Phi_{J=j}(l_1, l_2, \ldots, l_k, \ldots, l_K) \prod_{k=1}^{K} \left(\binom{n_k}{l_k} (R_k(t))^{l_k} (1 - R_k(t))^{s_{k_j}}\right)\right) \quad (11)$$

For instance, consider the parallel-series system, as depicted in Fig. 1. Assuming that type 1 components have $R_1(t) = e^{-t}$ and type 2 components have $R_2(t) = e^{-t^2}$. By applying Eq. 11, the probabilities of the system existing in various states are as follows:

$$Pr(J_t = 1) = \Phi(l_1, l_2)\binom{n_1}{l_1}(R_1(t))^{l_1}(1 - R_1(t))^{s_{1_1}}\binom{n_2}{l_2}(R_2(t))^{l_2}(1 - R_2(t))^{s_{2_1}}$$
$$= \frac{1}{4}\binom{2}{1}(e^{-t})^1(1 - e^{-t})^0\binom{2}{1}\left(e^{-t^2}\right)^1(1 - R_2(t))^0 = e^{-t-t^2} \quad (12)$$

$$Pr(J_t = 2) = 2e^{-t-t^2} - e^{-2t-2t^2} \quad (13)$$

Definition 3: let $J \in \{0, 1, \ldots, j, \ldots, m\}$ be the random variable of system's state. The reliability function of the MSS-BC at state j can be computed as follow:

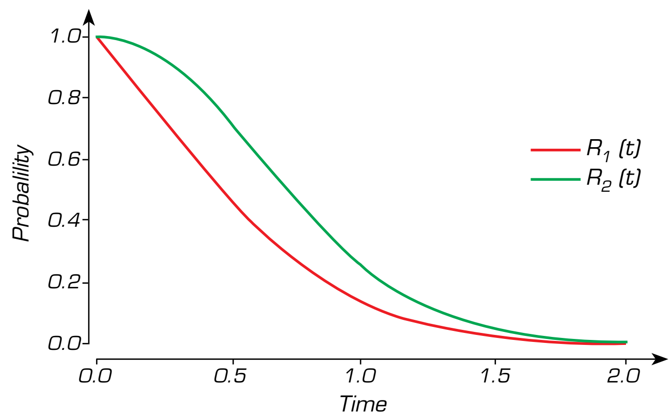
$$R_j(t) = \sum_{j=1}^{m} Pr(J_t = j) \quad (14)$$

For instance, the two types of reliability functions in the perfect and intermediate states can be computed using Eqs. 12–14 and plotted, as shown in Fig. 4.

Consequently, Eq. 11 effectively calculates the probability of the system operating in each state using the disjoint product form, removing the Bernoulli property. Equation 14 is utilized to determine the reliability of the MSS-BC in each state of the system.

Importance measure of MSS-BC

Reliability importance measures are crucial for evaluating industrial system security and managing risks. Analyzing reliability, importance, and sensitivity provides valuable insights for designers and helps technicians allocate resources effectively (Kuo and Zhu 2012; Mi *et al.* 2020; Zhou *et al.* 2019). In the following sections, three important measures based on the survival signature will be discussed.



Source: Elaborated by the authors.

Figure 4. Reliability functions of parallel-series MSS-BC at different states.

Birnbaum Importance measure

One of the most commonly used measures of significance is the BI index measure (Birnbaum 1968). The importance of component i is determined by the difference in reliability between a flawless component i and a system with a failed component i. The metric quantifies the probability that the system is in a condition where the operation of component i is crucial. The BI index measure with respect to the number of types $K \geq 2$ components $k \in \{1, 2, \dots, K\}$ and $l_k = 0, 1, \dots, n_k$ can be derived from Eq. 11 as follows:

$$I_i^{BI}(J_t = j) = \sum_{l_1=0}^{n_1} \cdots \sum_{l_K=0}^{n_{K-1}} \left(\Phi_{J=j}^{c_i=1}(l_1, l_2, \dots, l_K) - \Phi_{J=j}^{c_i=0}(l_1, l_2, \dots, l_K) \prod_{k=1}^{K} \left(\binom{n_k}{l_k} (R_k(t))^{l_k} (1 - R_k(t))^{s_{k_j}} \right) \right) \tag{15}$$

where $I_i^{BI}(J_t = j)$ is the BI index of component i at state $j \in \{1,2, \dots, m\}$ and time t. The BI index measure given in Eq. 15 is the effect of small modifications in component reliability on the overall system reliability. Therefore, it ranks components based on this effect.

Improvement potential measure

The IP index measure is a powerful tool for evaluating the potential impact of achieving complete reliability in a single system component (Aven and Nøkland 2010). It measures the maximum potential for increasing the reliability of a specific component by calculating the percentage difference between the reliability of a system with an ideal component and the reliability of the system with the actual component. The IP index measure with respect to number of types $K \geq 2$ components $k \in \{1, 2, \dots, K\}$ and $l_k = 0, 1, \dots, n_k$ can be obtained from Eq. 11 as follows:

$$I_i^{IP}(J_t = j) = \sum_{l_1=0}^{n_1} \cdots \sum_{l_K=0}^{n_{K-1}} \left(\Phi_{J=j}^{c_i=1}(l_1, l_2, \dots, l_K) - \Phi_{J=j}(l_1, l_2, \dots, l_K) \prod_{k=1}^{K} \left(\binom{n_k}{l_k} (R_k(t))^{l_k} (1 - R_k(t))^{s_{k_j}} \right) \right) \tag{16}$$

where $I_i^{IP}(J_t = j)$ is the IP index of component i at state $j \in \{1,2, \dots, m\}$ and time t. Furthermore, the IP index measure in Eq. 16 can be readily adapted to assess risk indices. In contrast, the Birnbaum measure is commonly applied during functions, while the IP is predominantly utilized in the structure stage (Vaisman and Sun 2021; Zaitseva and Levashenko 2013).

Risk reduction measure

The RR index measure establishes how potential failures or lapsed components affect system reliability. In other words, this measure may be used to identify system elements that are the best candidates for efforts leading to reducing the system risk (or improving safety) (Van der Borst and Schoonakker 2001). The RR index measure is the difference between the system reliability with the actual component and the system reliability with a failed component . The RR index measure with respect to the number of types $K \geq 2$ components $k \in \{1, 2, \ldots, K\}$ and $l_k = 0, 1, \ldots, n_k$ can be constructed from Eq. 11 as follows:

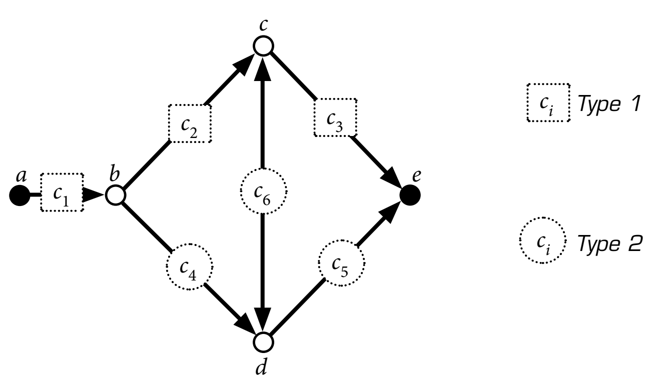
$$I_i^{RR}(J_t = j) = \sum_{l_1=0}^{n_1} \cdots \sum_{l_K=0}^{n_{K-1}} \left(\Phi_{J=j}(l_1, l_2, \ldots, l_K) - \Phi_{J=j}^{c_i=0}(l_1, l_2, \ldots, l_K) \prod_{k=1}^{K} \left(\binom{n_k}{l_k} \left(R_k(t)\right)^{l_k} \left(1 - R_k(t)\right)^{s_k} \right) \right) \quad (17)$$

where $I_i^{RR}(J_t = j)$ is the RR index of component i at state $j \in \{1,2,\ldots,m\}$ and time t. This importance measure in Eq. 17 is defined as potential failure space. This is of interest when planning different maintenance activities for example testing single components if a component is critical for the safe operation of the system.

NUMERICAL EXAMPLE

Model description

In this section, a numerical example is provided to illustrate the application of the suggested technique for determining the survival function of MSS-BC at a specific state, $J = j$, and estimating the reliability of MSS-BC. The three important measures based on the survival signature are also determined. Figure 5 depicts a simple bridge system with six binary-state components divided into two types: c_1, c_2, and c_3 belong to type 1, while c_4, c_5, and c_6 belong to type 2. This system has been referenced in several related works, such as Qin and Coolen (2022) and Yang *et al.* (2024b), for determining the survival function and reliability evaluation, offering an opportunity for comparison.



Source: Adapted from Coolen and Coolen-Maturi (2012).

Figure 5. A graph of a bridge system with two types of components.

For instance, it is essential to note that the failure time of type 1 components adheres to a Weibull distribution, i.e., $R_1(t) = e^{-\lambda_1 \frac{t^{\beta_1+1}}{\beta_1+1}}$ where $\lambda_1 = 0.5$, and $\beta_1 = 1.2$, while the failure time of type 2 components is in line with a linear exponential distribution, i.e., $R_2(t) = e^{-\left(\lambda_2 t + \frac{1}{2}\beta_2 t^2\right)}$ where $\lambda_2 = 0.7$, and $\beta_2 = 1.4$. In the following subsection, the disjoint product form and the survival signature will be utilized to construct the system's reliability function of MSS-BC in different states.

Reliability calculation of bridge system

To calculate the reliability of MSS-BC based on the survival signature, the system state is determined based on the disjoint product forms of minimal path sets. According to the proposed algorithm for the disjoint product form, the system state is denoted as $j = 0$ if no working path exists, $j = 1$ if the system reaches its maximum flow, and $j = 2, 3$, or 4 for other scenarios. The bridge system in Fig. 5 has four working minimal path sets, which are: $\mathcal{MP}_1 = \{c_1, c_2, c_3\}, \mathcal{MP}_2 = \{c_1, c_4, c_5\}, \mathcal{MP}_3 = \{c_1, c_2, c_5, c_6\}$ and $\mathcal{MP}_4 = \{c_1, c_3, c_4, c_6\}$. The disjoint product form of Fig. 5 can be given as follows:

Step 0: input all minimal path sets $\mathcal{MP}$ as follows:

$\mathcal{MP}_1 = \{c_1, c_2, c_3\}, \mathcal{MP}_2 = \{c_1, c_4, c_5\}, \mathcal{MP}_3 = \{c_1, c_2, c_5, c_6\}$ *and* $\mathcal{MP}_4 = \{c_1, c_3, c_4, c_6\}$

Step 1: for $\mathcal{MP}_1 = \{c_1, c_2, c_3\}$: it is obtained that $v_{1,1} = (1,1,1,0,0,0)$ and $s_{k_1} = 0$, $k = 1,2,3,4$.

Step 2: for $\mathcal{MP}_2 = \{c_1, c_4, c_5\}$:

let $\mathcal{D}_{2,0} = \{c_1, c_4, c_5\}$

Step 2.3: $\mathcal{MP}_1 - \mathcal{D}_{2,0} = \{c_2, c_3\}, then\ \mathcal{D}_{2,1} = \{c_1, \bar{c}_2, c_4, c_5\} and\ \mathcal{D}_{2,2} = \{c_1, c_2, \bar{c}_3, c_4, c_5\}$

then $v_{2,1} = (1,0,0,1,1,0)$ and $v_{2,2} = (1,1,0,1,1,0)$ and $s_{1_1} = s_{1_2} = 1$.

Step 2: for $\mathcal{MP}_3 = \{c_1, c_2, c_5, c_6\}$:

let $\mathcal{D}_{3,0} = \{c_1, c_2, c_5, c_6\}$

Step 2.3: $\mathcal{MP}_1 - \mathcal{D}_{3,0} = \{c_3\}, then\ \mathcal{D}_{3,1} = \{c_1, c_2, \bar{c}_3, c_5, c_6\}$

Step 2.3: $\mathcal{MP}_2 - \mathcal{D}_{3,1} = \{c_4\}, then\ \mathcal{D}_{3,2} = \{c_1, c_2, \bar{c}_3, \bar{c}_4, c_5, c_6\}$

then $v_{3,1} = (1,1,0,0,1,1)$ and $s_{1_1} = s_{2_1} = 1$.

Step 2: for $\mathcal{MP}_4 = \{c_1, c_3, c_4, c_6\}$:

let $\mathcal{D}_{4,0} = \{c_1, c_3, c_4, c_6\}$

Step 2.3: $\mathcal{MP}_2 - \mathcal{D}_{4,1} = \{c_5\}, then\ \mathcal{D}_{4,2} = \{c_1, \bar{c}_2, c_3, c_4, \bar{c}_5, c_6\}$

Step 2.3: $\mathcal{MP}_2 - \mathcal{D}_{4,1} = \{c_5\}, then\ \mathcal{D}_{4,2} = \{c_1, \bar{c}_2, c_3, c_4, \bar{c}_5, c_6\}$

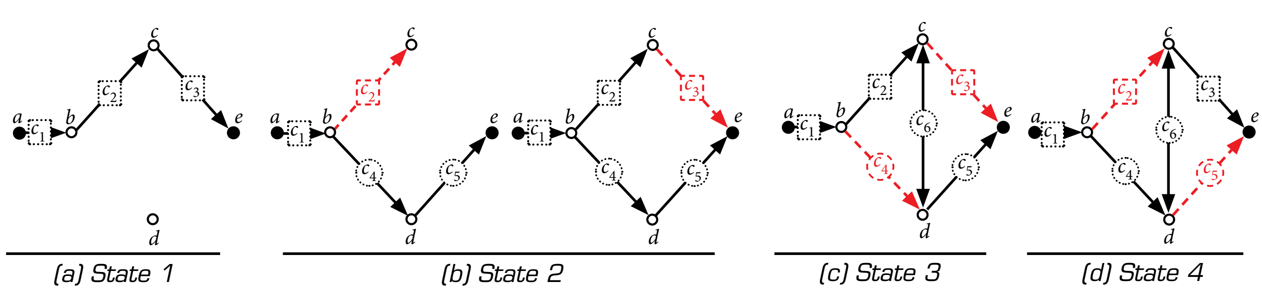
Step 2.2: $\mathcal{MP}_3 - \mathcal{D}_{4,2} = \emptyset, then\ \mathcal{D}_{4,3} = \{c_1, \bar{c}_2, c_3, c_4, \bar{c}_5, c_6\}$

then $v_{3,1} = (1,0,1,1,0,1)$ and $s_{1_1} = s_{2_1} = 1$.

The algorithm offers the advantage of assigning a vector to each state to represent a condition for extracting the required state using the survival signature. This technique eliminates the Bernoulli property. The structure function of the MSS-BC in Fig. 5 includes four operational states as follows:

$j = 1$: $v_{1,1} = (1,1,1,0,0,0)$ and $s_{1_1} = s_{2_1} = 0$,
$j = 2$: $v_{2,1} = (1,0,0,1,1,0)$, $v_{2,2} = (1,1,0,1,1,0)$ and $s_{1_1} = s_{1_2} = 1$,
$j = 3$: $v_{3,1} = (1,1,0,0,1,1)$ and $s_{1_1} = s_{2_1} = 1$,
$j = 4$: $v_{4,1} = (1,0,1,1,0,1)$ and $s_{1_1} = s_{2_1} = 1$.

The state vectors for each state can be used to calculate the survival signature for the four operational states as shown in Fig. 6. The survival signature $\Phi_{J=j}(l_1, l_2)$ for the five system states is listed in Table 3.



Source: Elaborated by the authors.

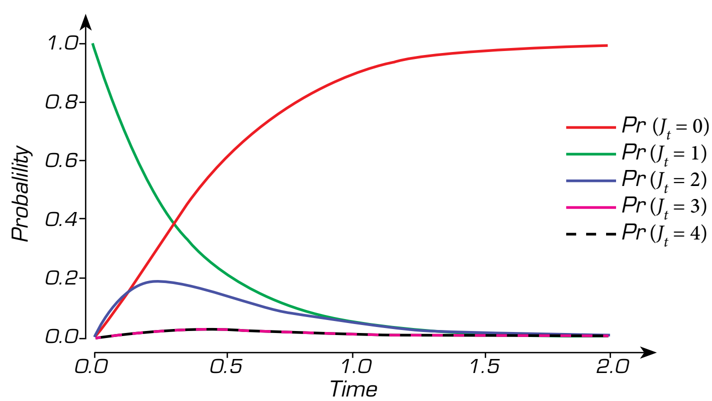
Figure 6. The MSS-BC bridge's operational states are represented in disjoint product forms where the red dashed line shows the negation component's state. The states are (a) the first operational state, (b) the second operational state, (c) the third operational state, and (d) the fourth operational state.

Table 3. Survival signatures of bridge MSS-BC for different states of system.

l_1	l_2	$\Phi_{J=j}(l_1, l_2)$				
		$j=0$	$j=1$	$j=2$	$j=3$	$j=4$
0	[0,1,2,3]	1	0	0	0	0
[1,2]	[0,1,3]	1	0	0	0	0
1	2	8/9	0	1/9	0	0
2	2	2/3	0	1/9	1/9	1/9
3	0	0	1	0	0	0
3	[1,2,3]	1	0	0	0	0

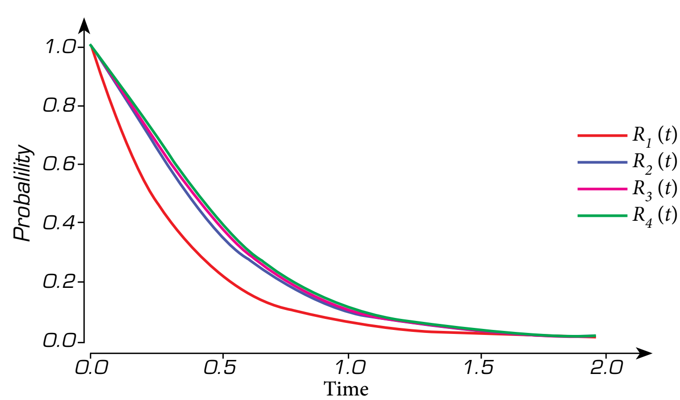
Source: Elaborated by the authors.

Then, by using Eq. 11, the probabilities of the system at various states can be displayed in Fig. 7. Furthermore, Fig. 8 illustrates the reliability functions at different states for a comprehensive comparison based on Eq. 14.



Source: Elaborated by the authors.

Figure 7. Bridge MSS-BC probabilities for various states at time.



Source: Elaborated by the authors.

Figure 8. Reliability functions of bridge MSS-BC at different states.

Importance measures of bridge system

In this section, the three importance measures for each state of the MSS-BC (shown in Fig. 5) are calculated using binary-state components. These measures are crucial as they quantify the disparity between the probabilities of the system functioning when component i is operational versus when it is not. To do this, the proposed algorithm for the disjoint product form determines each component's state (success or failure) for every operational state of the MSS-BC. These states are detailed in Table 4.

Table 4. State vectors of components of bridge MSS-BC.

c_i	$c_i = 1$				$c_i = 0$			
	$\boldsymbol{j} = 1$	$\boldsymbol{j} = 2$	$\boldsymbol{j} = 3$	$\boldsymbol{j} = 4$	$\boldsymbol{j} = 1$	$\boldsymbol{j} = 2$	$\boldsymbol{j} = 3$	$\boldsymbol{j} = 4$
c_1	(**1**,1,1,0,0,0)	(**1**,0,0,1,1,0), (**1**,1,0,1,1,0), $s_{1_1} = s_{1_2} = 1$	(**1**,1,0,0,1,1), $s_{1_1} = s_{1_2} = 1$	(**1**,0,1,1,0,1), $s_{1_1} = s_{1_2} = 1$	-	-	-	-
c_2	(1,**1**,1,0,0,0)	(1,**1**,0,1,1,0), $s_{1_1} = 1$	(1,**1**,0,0,1,1), $s_{1_1} = s_{1_2} = 1$	-	(1,**0**,0,1,1,0)	(1,**0**,1,1,0,1), $s_{2_1} = 1$	-	-
c_3	(1,1,**1**,0,0,0)	(1,0,**1**,1,1,0), $s_{1_1} = 1$	(1,0,**1**,1,0,1), $s_{1_1} = s_{1_2} = 1$	-	(1,0,**0**,1,1,0)	(1,1,**0**,0,1,1), $s_{2_1} = 1$	-	-
c_4	(1,0,0,**1**,1,0)	(1,1,1,**1**,0,0), $s_{2_1} = 1$	(1,0,1,**1**,0,1), $s_{1_1} = s_{1_2} = 1$	-	(1,1,1,**0**,0,0)	(1,1,0,**0**,1,1), $s_{1_1} = 1$	-	-
c_5	(1,0,0,1,**1**,0)	(1,1,0,1,**1**,0), $s_{2_1} = 1$	(1,1,0,0,**1**,1), $s_{1_1} = s_{1_2} = 1$	-	(1,1,1,0,**0**,0)	(1,0,1,1,**0**,1), $s_{1_1} = 1$	-	-
c_6	(1,1,1,0,0,**1**)	(1,0,0,1,1,**1**), (1,1,0,1,1,**1**), $s_{1_1} = s_{1_2} = 1$	(1,1,0,0,1,**1**), $s_{1_1} = s_{1_2} = 1$	(1,0,1,1,0,**1**), $s_{1_1} = s_{1_2} = 1$	(1,1,1,0,0,**0**)	(1,0,0,1,1,**0**), (1,1,0,1,1,**0**), $s_{1_1} = 1$	-	-

Source: Elaborated by the authors.

Bold type denotes the fixed state of the component in the system's state vector.

For instance, the survival signature of MSS-BC in Fig. 5, when component 6 operates and fails, can be determined based on the proposed algorithm for the disjoint product form. The survival signature of these subsystems can be represented as $\Phi_{J=j}^{c_6=1}(l_1, l_2)$ and $\Phi_{J=j}^{c_6=0}(l_1, l_2)$, and the results can be seen in Table 5.

Table 5. The survival signature of the two circumstances of component 6.

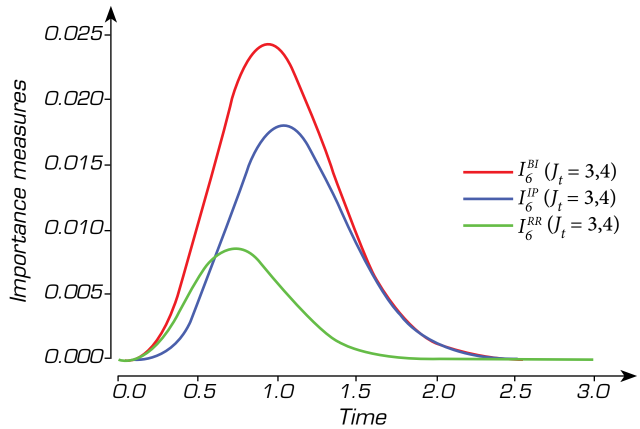
$\boldsymbol{l_1}$	$\boldsymbol{l_2}$	$\boldsymbol{\Phi_{J=j}^{c_6=1}(l_1, l_2)}$				$\boldsymbol{\Phi_{J=j}^{c_6=0}(l_1, l_2)}$			
		$\boldsymbol{j} = 1$	$\boldsymbol{j} = 2$	$\boldsymbol{j} = 3$	$\boldsymbol{j} = 4$	$\boldsymbol{j} = 1$	$\boldsymbol{j} = 2$	$\boldsymbol{j} = 3$	$\boldsymbol{j} = 4$
0	[0,1,2]	0	0	0	0	0	0	0	0
1	[0,1]	0	0	0	0	0	0	0	0
1	2	0	1/3	0	0	0	1/3	0	0
2	0	0	0	0	0	0	0	0	0
2	**1**	**0**	**0**	**1/6**	**1/6**	**0**	**0**	**0**	**0**
2	2	0	1/3	0	0	0	1/3	0	0
3	0	1	0	0	0	1	0	0	0
3	[1,2]	0	0	0	0	0	0	0	0

Source: Elaborated by the authors.

Bold type denotes the differences between the operation and failure of component 6.

Furthermore, the analytical approach can be used to compute the BI, IP, and RR measures of component 6 at states $j = 3$ *and* $j = 4$ of MSS-BC in Fig. 5. The results of this calculation can be shown in Fig. 9. For generality, in calculating the three importance index measures, failure times of the remaining components provide exact distribution parameters, e.g., $\lambda_1 = 0.5, \lambda_2 = 0.7, \beta_1 = 1.2$ and $\beta_2 = 1.4$, and time $t = 0.7$. time . The results are shown in Table 6.

Table 6 displays the values of the importance index measures for each state of each component of the MSS-BC in Fig. 5. It is important to note that certain importance index measures have resulted in negative values due to system state overlaps. In such cases, these negative values have been adjusted to zero. Moreover, the importance index measures for the overall MSS-BC are provided in the last three columns of Table 6, representing the sum of the importance index measures for each state of the MSS-BC.



Source: Elaborated by the authors.

Figure 9. The BI, IP, and RR measures of component 6 in the bridge system at state.

Table 6. The BI, IP, and RR measures of MSS-BC in Fig. 5.

c_i	$\Phi_{J=1}(l_1,l_2)$			$\Phi_{J=2}(l_1,l_2)$			$\Phi_{J=3}(l_1,l_2)$			$\Phi_{J=4}(l_1,l_2)$			$\Phi(l_1,l_2)$		
	$I_i^{BI}(J_t)$	$I_i^{IP}(J_t)$	$I_i^{RR}(J_t)$	$I_i^{BI}(J_t)$	$I_i^{IP}(J_t)$	$I_i^{RR}(J_t)$	$I_i^{BI}(J_t)$	$I_i^{IP}(J_t)$	$I_i^{RR}(J_t)$	$I_i^{BI}(J_t)$	$I_i^{IP}(J_t)$	$I_i^{RR}(J_t)$	$I_i^{BI}(J_t)$	$I_i^{IP}(J_t)$	$I_i^{RR}(J_t)$
c_1	0.8127	0.0800	0.7326	0.0353	0.0034	0.0319	0.0094	0.0009	0.0085	0.0094	0.0009	0.0085	0.8670	0.0854	0.7816
c_2	0.6423	0.0800	0.5622	-0.0700	-0.0151	-0.0549	0.0094	0.0009	0.0085	0.0000	-0.0085	0.0085	0.5817	0.0573	0.5244
c_3	0.6423	0.0800	0.5622	-0.0700	-0.0151	-0.0549	0.0094	0.0009	0.0085	0.0000	-0.0085	0.0085	0.5817	0.0573	0.5244
c_4	-0.3407	-0.3407	0.0000	0.3990	0.3822	0.0167	0.0196	0.0111	0.0085	0.0000	-0.0085	0.0085	0.0779	0.0440	0.0338
c_5	-0.3407	-0.3407	0.0000	0.3990	0.3822	0.0167	0.0196	0.0111	0.0085	0.0000	-0.0085	0.0085	0.0779	0.0440	0.0338
c_6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0196	0.0111	0.0085	0.0196	0.0111	0.0085	0.0392	0.0222	0.0170

Source: Elaborated by the authors.

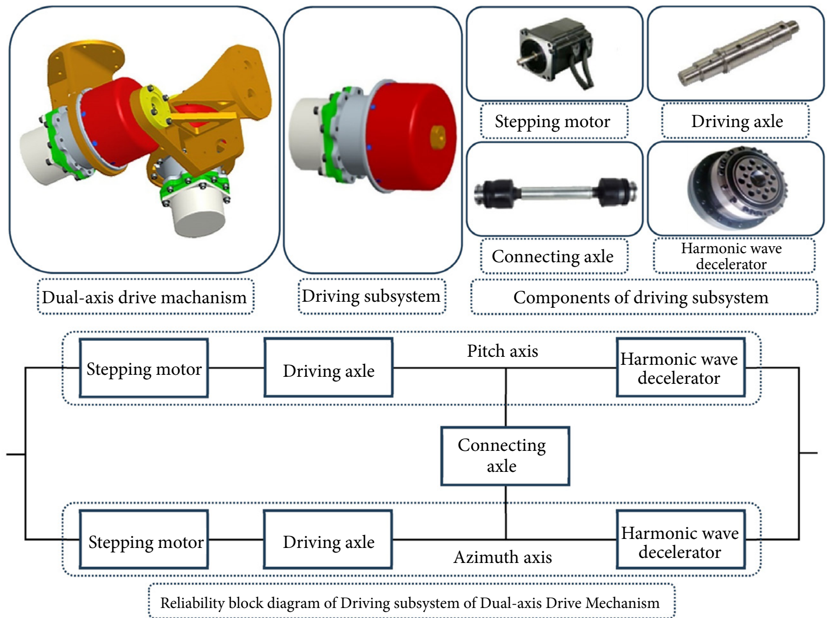
APPLICATION EXAMPLE

Model description

The driving subsystem is vital as electromechanical equipment widely utilized for dual-axis drive mechanisms in industrial engineering, particularly for intricate spatial motions. Its significance is evident when installed on a satellite to deploy an antenna. Since the equipment becomes unrepairable once the satellite is launched, its reliability is paramount. Any failure could jeopardize the success of the entire mission. The driving subsystem comprises four components: stepping motors 1 and 2, driving axles 1 and 2, connecting axle, and harmonic wave decelerators 1 and 2 (Yang *et al.* 2024b). The schematic diagram of the driving subsystem is depicted in Fig. 10.

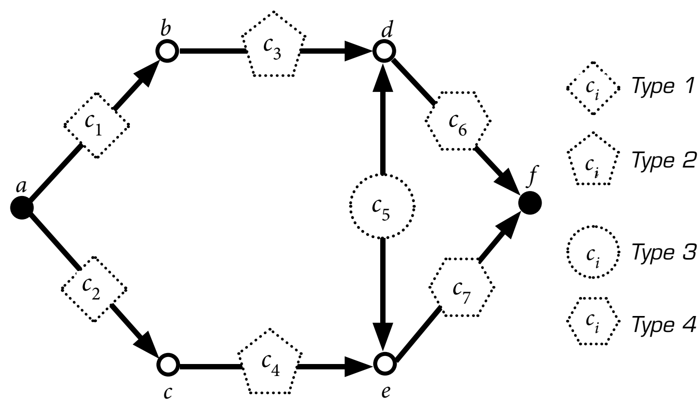
The two-terminal system's reliability quantifies the likelihood of successful data transfer from source to sink, determining the probability of data being effectively transmitted through non-failed connections between the source and sink points. For example, the complex system shown in Fig. 10 can be expressed as the mathematical notation graph $G = (V, E)$, where $V = \{a, b, \dots, f$ and $E = \{1, 2, \dots, 7\}$, representing the two-terminal graph. The driving subsystem comprises four key components: the steering motor (c_1 and c_2), the driving axle (c_3 And c_4), the connecting axle (c_5), and the harmonic wave decelerator (c_6 and c_7). The graph illustrating the driving subsystem is displayed in Fig. 11.

Furthermore, due to the limitations of the test data and the ambiguity in previous details, certain distribution parameters cannot be accurately determined and are instead represented as intervals. For MSS-BC in Fig. 11, the detailed parameter settings of the numerical simulation are summarized in Table 7.



Source: Adapted from Yang *et al.* (2024b).

Figure 10. Schematic and block diagram of driving subsystem.



Source: Elaborated by the authors.

Figure 11. A graph of driving subsystem.

Table 7. Parameter settings of components of the driving subsystem.

Components	Component type	Distribution type		Parameter setting
c_1 and c_2	1	Weibull	$R_i(t) = e^{-\lambda_i \frac{t^{\beta_i+1}}{\beta_i+1}}$	$\beta_1, \beta_2 \in [2{,}\times 10^6 4 \times 10^6]\, \lambda_1, \lambda_2 \in [0.1 \times 10^6, 2 \times 10^6]$,
c_3 and c_4	2	Linear-exponential	$R_i(t) = e^{-\left(\lambda_i t + \frac{1}{2}\beta_i t^2\right)}$	$\beta_3, \beta_4 \in [0.5{,}\times 10^6 2 \times 10^6]\ \lambda_3, \lambda_4 \in [0.2 \times 10^6, 3 \times 10^6]$,
c_5	3	Weibull	$R_i(t) = e^{-\lambda_i \frac{t^{\beta_i+1}}{\beta_i+1}}$	$\beta_5 \in [0.5{,}\times 10^6 2.6 \times 10^6]\, \lambda_5 \in [0.1 \times 10^6, 3 \times 10^6]$,
c_6 and c_7	4	Linear-exponential	$R_i(t) = e^{-\left(\lambda_i t + \frac{1}{2}\beta_i t^2\right)}$	$\beta_6, \beta_7 \in [0.1 \times 10^6 2 \times 10^6{,}\ \lambda_6, \lambda_7 \in [0.5 \times 10^6, 4 \times 10^6]$,

Source: Elaborated by the authors.

Reliability evaluation of driving subsystem

Consider the driving subsystem illustrated in Fig. 11, where the distinct product forms of minimal path sets determine the system state. The system features four functional paths: $\mathcal{MP}_1 = \{c_1, c_3, c_6\}, \mathcal{MP}_2 = \{c_2, c_4, c_7\}, \mathcal{MP}_3 = \{c_1, c_3, c_5, c_7\}$, and $\mathcal{MP}_4 = \{c_2, c_4, c_5, c_6\}$. According to the proposed algorithm for the disjoint product form, the system state is $j = 0$ if no functional path exists, $j = 1$ if the system achieves maximum flow, and $j = 2, 3,$ or 4 for other scenarios. The distinct product form of Fig. 11 can be formulated as follows:

1. Step 0: Input all minimal path sets $\mathcal{MP}$ as follows:
   $\mathcal{MP}_1 = \{c_1, c_3, c_6\}, \mathcal{MP}_2 = \{c_2, c_4, c_7\}, \mathcal{MP}_3 = \{c_1, c_3, c_5, c_7\}$ *and* $\mathcal{MP}_4 = \{c_2, c_4, c_5, c_6\}$
2. Step 1: For $\mathcal{MP}_1 = \{c_1, c_3, c_6\}$: it is obtained that $v_{1,1} = (1,0,1,0,0,1,0)$ and $s_{k_1} = 0, k = 1,2,3,4$.
3. Step 2: For $\mathcal{MP}_2 = \{c_2, c_4, c_7\}$:
   let $\mathcal{D}_{2,0} = \{c_2, c_4, c_7\}$
   Step 2.3: $\mathcal{MP}_1 - \mathcal{D}_{2,0} = \{c_1, c_3, c_6\}, then\ \mathcal{D}_{2,1} = \{\bar{c}_1, c_2, c_4, c_7\}, \mathcal{D}_{2,2} = \{c_1, c_2, \bar{c}_3, c_4, c_7\} and\ \mathcal{D}_{2,3} = \{c_1, c_2, c_3, c_4, \bar{c}_6, c_7\}$,
   then $v_{2,1} = (0,1,0,1,0,0,1)$, $v_{2,2} = (1,1,0,1,0,0,1)$ and $v_{2,3} = (1,1,1,1,0,0,1)$, *and* $s_{1_1} = s_{2_2} = s_{4_3} = 1$.
4. Step 2: For $\mathcal{MP}_3 = \{c_1, c_3, c_5, c_7\}$:
   let $\mathcal{D}_{3,0} = \{c_1, c_3, c_5, c_7\}$
   Step 2.3: $\mathcal{MP}_1 - \mathcal{D}_{3,0} = \{c_6\}, then\ \mathcal{D}_{3,1} = \{c_1, c_3, c_5, \bar{c}_6, c_7\}$
   Step 2.3: $\mathcal{MP}_2 - \mathcal{D}_{3,1} = \{c_2, c_4\}, then\ \mathcal{D}_{3,2} = \{c_1, \bar{c}_2, c_3, c_5, \bar{c}_6, c_7\}$ and $\mathcal{D}_{3,3} = \{c_1, c_2, c_3, \bar{c}_4, c_5, \bar{c}_6, c_7\}$
   then $v_{3,1} = (1,0,1,0,1,0,1)$, and $v_{3,2} = (1,1,1,0,1,0,1)$, *and* $s_{1_1} = s_{4_1} = s_{2_2} = s_{4_2} = 1$.
5. Step 2: For $\mathcal{MP}_4 = \{c_2, c_4, c_5, c_6\}$:
   let $\mathcal{D}_{4,0} = \{c_2, c_4, c_5, c_6\}$
   Step 2.3: $\mathcal{MP}_1 - \mathcal{D}_{4,0} = \{c_1, c_3\}, then\ \mathcal{D}_{4,1} = \{\bar{c}_1, c_2, c_4, c_5, c_6\}$ and $\mathcal{D}_{4,2} = \{c_1, c_2, \bar{c}_3, c_4, c_5, c_6\}$
   Step 2.3: $\mathcal{MP}_2 - \mathcal{D}_{4,1} = \{c_7\}, then\ \mathcal{D}_{4,3} = \{\bar{c}_1, c_2, c_4, c_5, c_6, \bar{c}_7\}$ and $\mathcal{MP}_2 - \mathcal{D}_{4,2} = \{c_7\}, then\ \mathcal{D}_{4,4} = \{c_1, c_2, \bar{c}_3, c_4, c_5, c_6, \bar{c}_7\}$
   Step 2.2: $\mathcal{MP}_3 - \mathcal{D}_{4,3} = \emptyset, then\ \mathcal{D}_{4,5} = \{\bar{c}_1, c_2, c_4, c_5, c_6, \bar{c}_7\}$ and $\mathcal{MP}_2 - \mathcal{D}_{4,2} = \emptyset, then\ \mathcal{D}_{4,6} = \{c_1, c_2, \bar{c}_3, c_4, c_5, c_6, \bar{c}_7\}$
   then $v_{4,1} = (0,1,0,1,1,1,0)$, and $v_{4,2} = (1,1,0,1,1,1,0)$, *and* $s_{1_1} = s_{4_1} = s_{2_2} = s_{4_2} = 1$.

The structure function of the MSS-BC in Fig. 11 includes four operational states as follows:

$j = 1$: $v_{1,1} = (1,0,1,0,0,1,0)$ and $s_{k_1} = 0, k = 1,2,3,4,$

$j = 2$: $v_{2,1} = (0,1,0,1,0,0,1)$, $v_{2,2} = (1,1,0,1,0,0,1)$ and $v_{2,3} = (1,1,1,1,0,0,1)$, *and* $s_{1_1} = s_{2_2} = s_{4_3} = 1,$

$j = 3$: $v_{3,1} = (1,0,1,0,1,0,1)$, and $v_{3,2} = (1,1,1,0,1,0,1)$, *and* $s_{1_1} = s_{4_1} = s_{2_2} = s_{4_2} = 1,$

$j = 4$: $v_{4,1} = (0,1,0,1,1,1,0)$, and $v_{4,2} = (1,1,0,1,1,1,0)$, *and* $s_{1_1} = s_{4_1} = s_{2_2} = s_{4_2} = 1.$

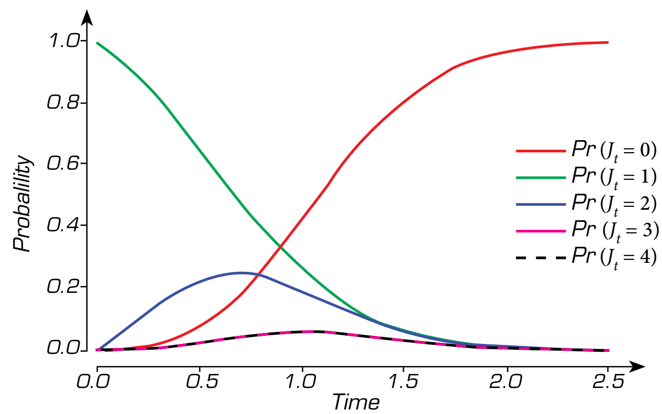
A critical study thoroughly evaluates the reliability of the driving subsystem throughout its lifespan to achieve this goal. A comprehensive analysis using the proposed method was conducted. Firstly, the vectors that define the states of the MSS-BC are calculated according to the proposed algorithm for the disjoint product form, as in the solution above. Secondly, the survival signature $\Phi_{J=j}(l_1, l_2, l_3, l_4)$ of MSS-BC for all state combinations of the driving subsystem is calculated, as depicted in Table 8.

Table 8. Survival signatures of driving subsystem for different states of system.

l_1	l_2	l_3	l_4	$\Phi_{J=j}(l_1, l_2, l_3, l_4)$				
				$j = 0$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
0	[0,1,2]	[0,1]	[0,1,2]	1	0	0	0	0
[1,2]	0	[0,1]	[0,1,2]	1	0	0	0	0
[1,2]	[1,2]	[0,1]	[0,2]	1	0	0	0	0
1	2	[0,1]	1	1	0	0	0	0
2	2	1	1	1	0	0	0	0
1	1	0	1	3/4	1/8	1/8	0	0
1	1	1	1	3/4	0	0	1/8	1/8
2	1	0	1	3/4	0	1/4	0	0
2	1	1	1	1/2	0	0	1/4	1/4
2	2	0	1	1/2	0	1/2	0	0

Source: Elaborated by the authors.

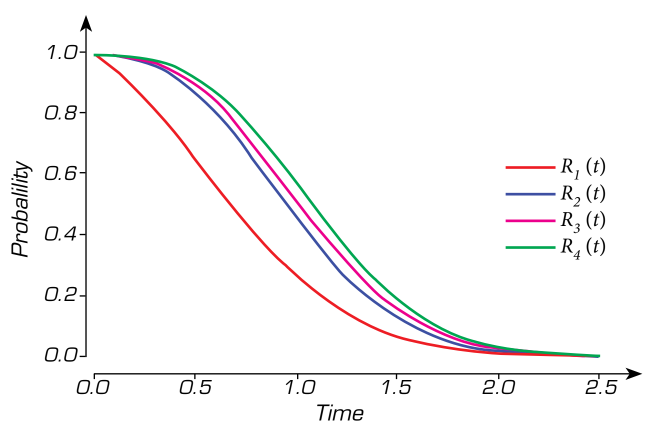
Subsequently, based on Eq. 11, the probabilities of the MSS-BC being in different states are estimated, providing an insightful analytical solution. Provided that the failure times of the components correspond to exact distribution parameters, e.g., $\lambda_1 = \lambda_2 = 0.10, \lambda_3 = \lambda_4 = 0.20, \lambda_5 = 0.30, \lambda_6 = \lambda_7 = 0.25, \beta_1 = \beta_2 = 2.10, \beta_3 = \beta_4 = 0.90, \beta_5 = 2.30$ and $\beta_6 = \beta_7 = 0.95$. The probabilities of the system at various states are visually presented in Fig. 12.



Source: Elaborated by the authors.

Figure 12. The probabilities of the driving subsystem for different states at time *t*.

In Yang *et al*. (2024b), the analysis of system states was based on the number of components in the minimal path, which only defines two states for the system, as noted in Qin and Coolen (2022). In contrast, the proposed technique examines the system states for each minimal path separately, leading to a more accurate representation of the system's operational states. This approach defines four states for the same system studied in Yang *et al*. (2024b). In other hand, the system's reliability function for various states can be obtained using Eq. 14, as illustrated in Fig. 13.



Source: Elaborated by the authors.

Figure 13. Reliability functions the driving subsystem for different states at time *t*.

Importance measure of the driving subsystem

The importance index measures are essential for evaluating the performance of the driving subsystem. These measures assess the impact of each component's state on the system's functionality. By utilizing the proposed algorithm for the disjoint product form, the state of each component (success or failure) at each state of the MSS-BC depicted in Fig. 11 were determined. These crucial state vectors of components are displayed in Table 9, providing valuable insights into the subsystem's performance at different time intervals.

Table 9. State vector of components of driving subsystem.

c_i	$c_i = 1$				$c_i = 0$			
	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
c_1	(**1**,0,1,0,0,1,0)	(**1**,1,0,1,0,0,1), (**1**,1,1,1,0,0,1), $s_{2_1} = s_{4_2} = 1$	(**1**,0,1,0,1,0,1), (**1**,1,1,0,1,0,1), $s_{1_1} = s_{4_1} = s_{2_2} = s_{4_2} = 1$	(**1**,1,0,1,1,1,0), $s_{2_1} = s_{4_2} = 1$	(**0**,1,0,1,0,0,1)	(**0**,1,0,1,1,1,0), $s_{4_1} = 1$	-	-
c_2	(0,**1**,0,1,0,0,1)	(1,**1**,1,0,0,1,0), (1,**1**,1,1,0,1,0), $s_{2_1} = s_{4_2} = 1$	(0,**1**,0,1,1,1,0), (1,**1**,0,1,1,1,0), $s_{1_1} = s_{4_1} = s_{2_2} = s_{4_2} = 1$	(1,**1**,1,0,1,0,1), $s_{2_1} = s_{4_2} = 1$	(1,**0**,1,0,0,1,0)	(1,**0**,1,0,1,0,1), $s_{4_1} = 1$	-	-
c_3	(1,0,**1**,0,0,1,0)	(0,1,**1**,1,0,0,1), (1,1,**1**,1,0,0,1), $s_{2_1} = s_{4_2} = 1$	(1,0,**1**,0,1,0,1), (1,1,**1**,0,1,0,1), $s_{1_1} = s_{4_1} = s_{2_2} = s_{4_2} = 1$	(0,1,**1**,1,1,1,0), $s_{1_1} = s_{4_1} = 1$	(0,1,**0**,1,0,0,1)	(0,1,**0**,1,1,1,0), $s_{4_1} = 1$	-	-
c_4	(0,1,0,**1**,0,0,1)	(1,0,1,**1**,0,1,0), (1,1,1,**1**,0,1,0), $s_{2_1} = s_{4_2} = 1$	(0,1,0,**1**,1,1,0), (1,1,0,**1**,1,1,0), $s_{1_1} = s_{4_1} = s_{2_2} = s_{4_2} = 1$	(1,0,1,**1**,1,0,1), $s_{1_1} = s_{4_1} = 1$	(1,0,1,**0**,0,1,0)	(1,0,1,**0**,1,0,1), $s_{4_1} = 1$	-	-
c_5	(1,0,1,0,**1**,1,0)	(0,1,0,1,**1**,0,1), (1,1,0,1,**1**,0,1), (1,1,1,1,**1**,0,1), $s_{1_1} = s_{2_2} = s_{4_3} = 1$	(1,0,1,0,**1**,0,1), (1,1,1,0,**1**,0,1), $s_{1_1} = s_{4_1} = s_{2_2} = s_{4_2} = 1$	(0,1,0,1,**1**,1,0), (1,1,0,1,**1**,1,0), $s_{1_1} = s_{4_1} = s_{2_2} = s_{4_2} = 1$	(1,0,1,0,**0**,1,0)	(0,1,0,1,**0**,0,1), (1,1,0,1,**0**,0,1), (1,1,1,1,**0**,0,1), $s_{1_1} = s_{4_1} = s_{2_2} = s_{4_2} = 1$	-	-
c_6	(1,0,1,0,0,**1**,0)	(0,1,0,1,0,**1**,1), (1,1,0,1,0,**1**,1), $s_{2_1} = s_{4_2} = 1$	(0,1,0,1,1,**1**,0), (1,1,0,1,1,**1**,0), $s_{1_1} = s_{4_1} = s_{2_2} = s_{4_2} = 1$	-	(0,1,0,1,0,**0**,1)	(1,0,1,0,1,**0**,1), (1,1,1,0,1,**0**,1), $s_{1_1} = s_{2_2} = 1$	-	-
c_7	(0,1,0,1,0,0,**1**)	(1,0,1,0,0,1,**1**), (1,1,1,0,0,1,**1**), $s_{2_1} = s_{4_2} = 1$	(1,0,1,0,1,0,**1**), (1,1,1,0,1,0,**1**), $s_{1_1} = s_{4_1} = s_{2_2} = s_{4_2} = 1$	-	(1,0,1,0,0,1,**0**)	(0,1,0,1,1,1,**0**), (1,1,0,1,1,1,**0**), $s_{1_1} = s_{2_2} = 1$	-	-

Source: Elaborated by the authors.

Bold type denotes the fixed state of the component in the system's state vector.

For instance, when component 5 is operational, or in the event of a failure at time t, the survival signatures for the driving subsystem are denoted as $\Phi_{J=j}^{c_5=1}(l_1, l_2, l_3, l_4)$ and $\Phi_{J=j}^{c_5=0}(l_1, l_2, l_3, l_4)$. The survival signatures when component 5 is working and when it fails are determined in Table 10.

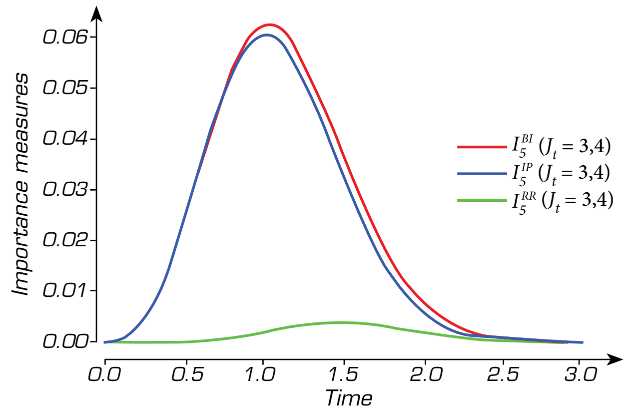
Table 10. The survival signatures when component 5 is working and failure.

l_1	l_2	l_4	$\Phi_{J=j}^{c_5=1}(l_1, l_2, l_3, l_4)$					$\Phi_{J=j}^{c_5=0}(l_1, l_2, l_3, l_4)$				
			$j = 0$	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 0$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
0	[0,1,2]	[0,1,2]	1	0	0	0	0	1	0	0	0	0
[1,2]	0	[0,1,2]	1	0	0	0	0	1	0	0	0	0
[1,2]	[1,2]	[0,2]	1	0	0	0	0	1	0	0	0	0
1	2	1	1	0	0	0	0	1	0	0	0	0
1	**1**	**1**	**1/2**	**1/8**	**1/8**	**1/8**	**1/8**	**3/4**	**1/8**	**1/8**	**0**	**0**
2	**1**	**1**	**3/4**	**0**	**1/4**	**1/4**	**1/4**	**3/4**	**0**	**1/4**	**0**	**0**
2	2	1	1/2	0	1/2	0	0	1/2	0	1/2	0	0

Source: Elaborated by the authors.

Bold type denotes the differences between the operation and failure of component 5.

Suppose a precise distribution parameter for the component failure times, such as $\lambda_1 = \lambda_2 = 0.10, \lambda_3 = \lambda_4 = 0.20$, $\lambda_5 = 0.30, \lambda_6 = \lambda_7 = 0.25, \beta_1 = \beta_2 = 2.10, \beta_3 = \beta_4 = 0.90, \beta_5 = 2.30$ and $\beta_6 = \beta_7 = 0.95$. In that case, the relative BI, IP, and RR measures of component 5 can be calculated at states $j = 3$ and $j = 4$ of MSS-BC in Fig. 11. These results are visualized in Fig. 14. Moreover, for a more comprehensive analysis, the results of calculating the three importance index measures of the remaining components' failure times at a specific time $t = 1.1$ are also included in Table 11.



Source: Elaborated by the authors.

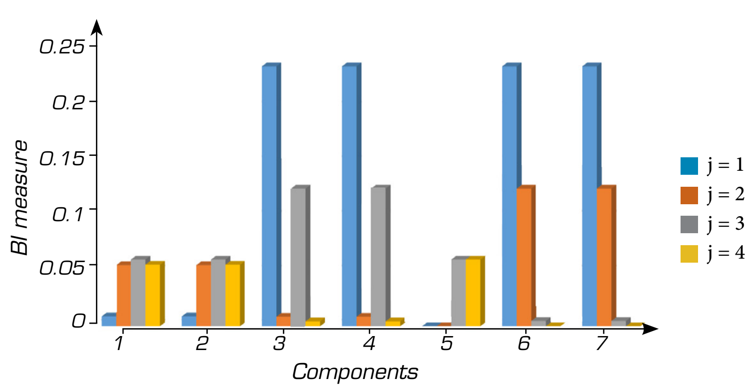
Figure 14. The BI, IP, and RR measures of component 5 in the driving subsystem at state.

Table 11. The BI, IP, and RR measures of MSS-BC in Fig. 6.

c_i	$\varphi_{J=1}(l_1,l_2,l_3,l_4)$			$\varphi_{J=2}(l_1,l_2,l_3,l_4)$			$\varphi_{J=3}(l_1,l_2,l_3,l_4)$			$\varphi_{J=4}(l_1,l_2,l_3,l_4)$			$\varphi(l_1,l_2,l_3,l_4)$		
	$I_i^{BI}(J_t)$	$I_i^{IP}(J_t)$	$I_i^{RR}(J_t)$	$I_i^{BI}(J_t)$	$I_i^{IP}(J_t)$	$I_i^{RR}(J_t)$	$I_i^{BI}(J_t)$	$I_i^{IP}(J_t)$	$I_i^{RR}(J_t)$	$I_i^{BI}(J_t)$	$I_i^{IP}(J_t)$	$I_i^{RR}(J_t)$	$I_i^{BI}(J_t)$	$I_i^{IP}(J_t)$	$I_i^{RR}(J_t)$
c_1	0.0091	0.0091	0.0000	0.0563	-0.0019	0.0582	0.0614	0.0026	0.0588	0.0567	-0.0021	0.0588	0.1835	0.0077	0.1758
c_2	0.0091	0.0091	0.0000	0.0563	-0.0019	0.0582	0.0614	0.0026	0.0588	0.0567	-0.0021	0.0588	0.1835	0.0077	0.1758
c_3	0.2382	0.2382	0.0000	0.0088	-0.0494	0.0582	0.1264	0.0675	0.0588	0.0045	-0.0543	0.0588	0.3779	0.2020	0.1758
c_4	0.2382	0.2382	0.0000	0.0088	-0.0494	0.0582	0.1264	0.0675	0.0588	0.0045	-0.0543	0.0588	0.3779	0.2020	0.1758
c_5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0614	0.0026	0.0588	0.0614	0.0026	0.0588	0.1228	0.0052	0.1176
c_6	0.2382	0.2382	0.0000	0.1264	0.0721	0.0543	0.0048	-0.0539	0.0588	0.0000	-0.0588	0.0588	0.3694	0.1974	0.1719
c_7	0.2382	0.2382	0.0000	0.1264	0.0721	0.0543	0.0048	-0.0539	0.0588	0.0000	-0.0588	0.0588	0.3694	0.1974	0.1719

Source: Elaborated by the authors.

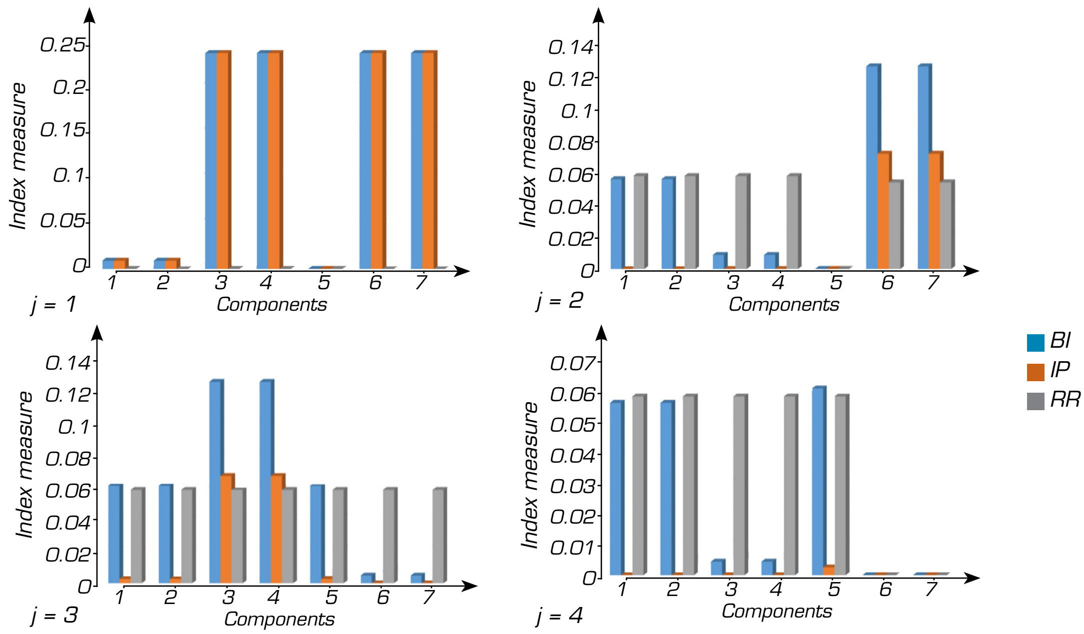
Table 11 provides a comprehensive breakdown of the importance index measures for each state of the MSS-BC and each component of the MSS-BC. Negative importance index measures resulting from system state overlap were treated as zero in the importance index measures. The last three columns in Table 11 represent the total importance index measures for the entire MSS-BC, achieved by summing the importance index measures for each state of the MSS-BC. Additionally, Fig. 15 illustrates



Source: Elaborated by the authors.

Figure 15. BI measure for each component of each state of driving subsystem.

the BI measure for each component of each state of the MSS-BC. To underscore the significance of the dependencies of the three importance index measures for every state of the driving subsystem, a comparison of BI, IP, and RR measures for each component of each state of the driving subsystem was conducted, as depicted in Fig. 16.



Source: Elaborated by the authors.

Figure 16. The BI, IP, and RR measures for each component of the driving subsystem in each state.

Certain components exhibit the IP and RR in specific states of the driving subsystem. In engineering applications, if these improvements and RRs are not properly addressed, they could lead to overly risky reliability estimation results, posing a potential danger. The importance index measures are critical for a satellite-based driving subsystem, where reliability is of the utmost concern in safety-critical fields such as aerospace engineering.

CONCLUSION

This paper presents a new reliability assessment approach for MSS-BC, based on disjoint product forms of minimal path sets and survival signatures. It also defines an MSS-BC model based on the disjoint product forms of minimal path sets to form states with the number of minimal paths required for system operation. The Bernoulli property was eliminated based on the survival signature and disjoint product forms. For this reason, the formula for computing the reliability function was updated based on a vector representing the state of the negated components. Additionally, the paper presents methods for the BI, IP, and RR measures based on disjoint product forms of minimal path sets and survival signatures. The method proved its accuracy and effectiveness by studying a numerical model. An applied model was then studied, and data was presented, showing its engineering and practical benefits.

The complexity of the proposed method increases with the number of minimal paths. As the number of minimal path sets in a system grows, the computational complexity also rises. This issue can be addressed by examining subsystems. The application of MSS-BC reliability is especially relevant in various fields, such as engineering, telecommunications, and transportation. In these systems, the overall system can operate at multiple performance levels while individual components are considered binary (either functioning or failed).

For future development in this area, expanding the methodology to include multi-state components in MSS-BC would be beneficial. This expansion would greatly enhance the applicability of the approach. Additionally, it would be worthwhile to investigate how different failure distributions affect structural importance measures. Another promising direction for future development is to develop general techniques for computational methods, statistical inference methods, detailed modeling of component state change processes, and decision support for inspection and maintenance. It is extremely valuable to ensure that these developments are closely tied to real-world applications to maximize their functional relevance in the future.

CONFLICT OF INTEREST

Nothing to declare.

AUTHORS' CONTRIBUTION

Conceptualization: Mutar EK; **Methodology:** Mutar EK; **Software:** Mutar EK; **Validation:** Mutar EK and Hassan ZAH; **Formal analysis:** Mutar EK and Hassan ZAH; **Data Curation:** Mutar EK; **Writing - Original Draft:** Mutar EK; **Writing - Review & Editing:** Mutar EK and Hassan ZAH; **Supervision:** Hassan ZAH; **Final approval:** Mutar EK.

DATA AVAILABILITY STATEMENT

All data sets were generated or analyzed in the current study.

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