

LQG-Based Model Predictive Control of the VLS-1 Attitude at Max Dynamic Pressure

Antonio Silveira^{1,*} , Marco Sagliano² , David Seelbinder³ , Stephan Theil³ , João Viana da Fonseca Neto⁴ , Rufin Azonsivo¹ , Matheus Moraes da Silva¹ , Marcelo Lima¹ , Daniel Silva¹ , Lucas Sodré¹ , Marcos Seruffo¹ 

1. Universidade Federal do Pará  – Programa de Pós-Graduação em Engenharia Elétrica – Belém/PA – Brazil.

2. University of Bologna  – Department of Industrial Engineering – Bologna – Italy.

3. German Aerospace Center  – Department of Guidance, Navigation and Control – Bremen – Germany.

4. Universidade Federal do Maranhão  – Programa de Pós-Graduação em Engenharia Aeroespacial – São Luís/MA – Brazil.

*Corresponding author: asilveira@ufpa.br

ABSTRACT

This paper presents the main results of a case study on the attitude control of a satellite launch vehicle using a novel model predictive control (MPC). Design and simulations were based on the Brazilian VLS-1 launcher during the maximum dynamic pressure regime. Such a model was characterized by a 16th-order state-space realization with asymmetrical and coupled attitude dynamics, bending modes, non-collocated sensor-actuator, actuator saturation, wind disturbances, and sensor noise. The idea was to recreate, from a control theory perspective, a scenario similar to the 1999 explosion of the VLS-1 V02, exploring the thesis that the intense vibratory regime could have contributed to the failure. It is demonstrated that when non-collocated sensor-actuator or measurement noise is introduced into the simulations, the control-loop performance degrades and stability may be lost with proportional-integral-derivative-based controllers. To overcome such problems, a new MPC technology is proposed, predictive minimum variance control with full state-feedback. The adopted methodology inherits the advantages of linear quadratic Gaussian with loop-transfer recovery and adds the enhancements of MPC to mitigate the effects of time delays. The results were compared to those obtained by applying a functional approximation of the PI-D controller once designed for the real VLS-1 launcher.

Keywords: Satellite launch vehicle; Model predictive control; Attitude control; Guidance, navigation & control; VLS-1.

INTRODUCTION

In this work, a novel model predictive control (MPC) method is proposed and investigated for the problem of attitude control of a satellite launch vehicle based on the Brazilian rocket known as VLS-1. In this study, the thesis presented by Palmerio (2017, p. 29), that the failure of the VLS-1 launched in 1999 could have been caused by the intense vibratory regime, is investigated. The considered simulation model was based on the maximum dynamic pressure (Max Q) state-space realization described by Ramos (2011, p. 99-101), having three inputs and three outputs, 16 state variables, coupled dynamics, with added bending modes, actuator saturation, non-collocated sensor-actuator, wind, and measurement noise disturbances.

The VLS-1 was the first Brazilian satellite launch vehicle project, having three prototypes built and identified as V01, V02, and V03. A fourth version was never finalized, and the project was canceled in 2016 (Silva *et al.* 2019). The downfall of the VLS-1 project occurred when the V03 exploded in 2003, 3 days before the launch date, while the vehicle was still inside the assembly building, destroying the facility and killing 21 people who were working at the site.

Received: Nov. 15, 2025 | **Accepted:** June 26, 2026

Peer Review History: Single Blind Peer Review.

Section editor: Luiz Martins-Filho 

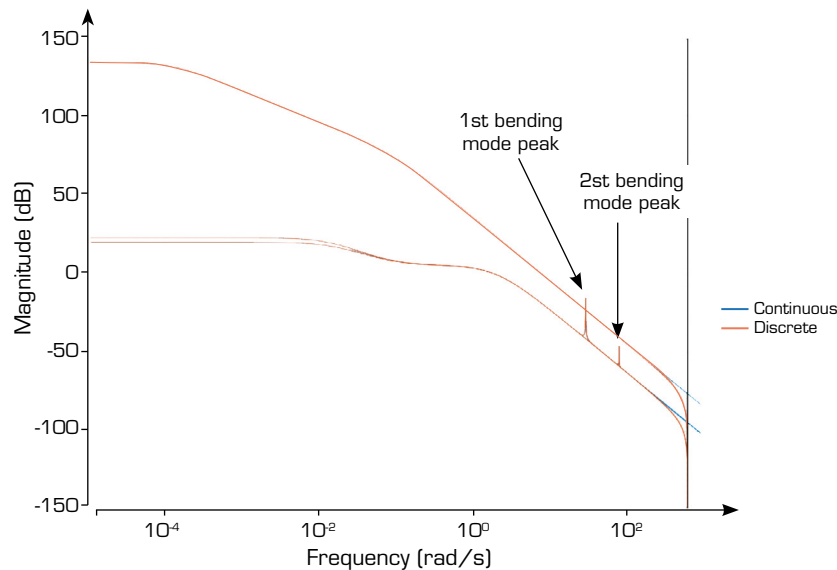


The V01 was launched in 1997, but one of its four boosters of the 1st-stage did not ignite, and the vehicle drifted off-course, generating an increased sideslip angle that led the vehicle to an unbearable aerodynamic stress and rupture, 26 seconds after liftoff (Palmerio 2017). The V02 was launched in 1999 and flew for almost 56 seconds until the engine of its 2nd-stage exploded after ignition (Retro Space HD 2022). The V02 endured the Max Q regime, and valuable data became available to enhance the knowledge of the VLS-1 during the intense vibratory regime.

The possible technical cause for the V02 engine explosion was detailed in Palmerio (2017, p. 138). Nonetheless, one thesis added by Palmerio (2017, p. 29) was that the intense vibratory regime could have caused the accident. It was remarked that no conclusive proof has been given to support this thesis.

The V02 prototype used attitude controllers structured as proportional-integral “minus” derivative (PI-D) (Ramos 2011; Silva *et al.* 2019), which avoids the “derivative kick”, which is an impulsive derivative action due to step-like changes in the attitude references. However, PI-D still differentiates noise or attitude angle vibrations and oscillations with respect to time, amplifying these values through the feedback loop to the actuators, leading to excessive control signal chattering and excitation of the bending modes, increased vibrations, and structural damage to the vehicle. Thus, from the control theory viewpoint, the thesis raised in Palmerio (2017) is plausible.

The VLS-1 was addressed in the literature as having two bending modes, near 30 and 80 rad/s (Fig. 1). However, the attitude PI-D controllers’ tuning and analysis procedures for the VLS-1 were based on simplified 2nd-order model approximations without the bending modes (Silva *et al.* 2019).



Source: Elaborated by the authors.

Figure 1. VLS-1 principal gains, remarking the two bending modes at 29.5 and 80.4 rad/s.

In Silva *et al.* (2019), the effects of the bending modes were not taken into consideration, as the authors assumed an ideal attenuation by notch filters. Yamada *et al.* (2024) revisited the attitude control problem of the VLS-1 using a control technology similar to the linear quadratic Gaussian (LQG) with loop-transfer-recovery (LTR). They considered the bending modes and, despite stating that the bending modes had been attenuated, in Yamada *et al.* (2024), the effects of these modes through the feedback loop were still noticeable in the control signals. In their simulations, no vibratory effects (i.e., noise) in the attitude measurements were considered, which could eventually increase the problem. Nonetheless, their LQG, based on a more complex design model and controller, considerably enhanced the robustness and performance of the VLS-1 attitude control.

Following the footsteps of Yamada *et al.* (2024), an LQG/LTR-based predictive minimum variance control (PMVC) with full state-feedback is proposed in this work for the first time. The PMVC is a multiple-input multiple-output (MIMO) MPC

technology previously addressed to deal with output feedback, focusing on minimizing both the tracking error and the control signals' power (Silveira *et al.* 2020). In this new approach, the PMVC algorithm is enhanced to full state-feedback to address the minimization of the whole set of state variables at once (Stevens *et al.* 2016), including, naturally and systematically, the bending modes minimization and stability to non-collocated sensor-actuator delays.

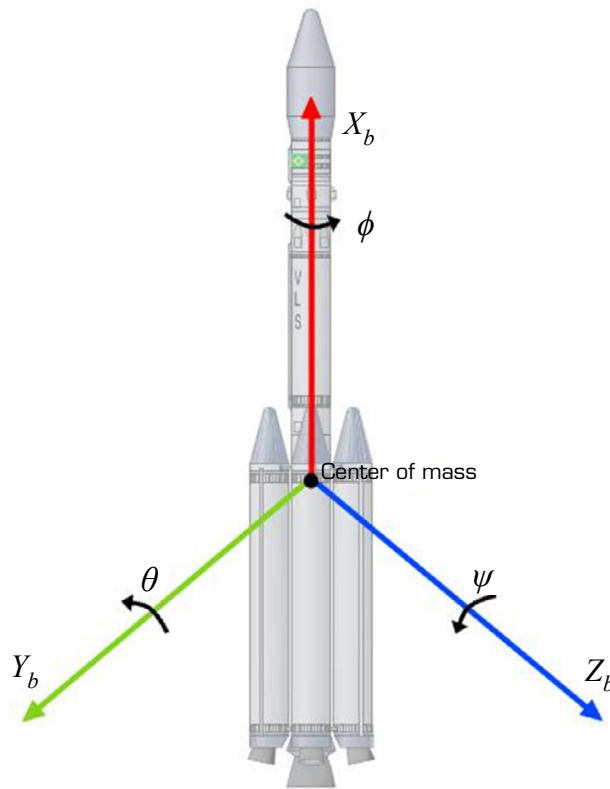
Beyond this introductory part, this work is organized as follows: next, the methodology is presented, comprising the VLS-1 simulation model, the baseline PI-D and its design technique, the PMVC with full state-feedback proposition, and relative stability analysis techniques to assess the control-loop robustness for the multivariate PMVC. This is followed by the simulation results, along with the discussions. Finally, conclusions and ideas for future work are drawn.

METHODOLOGY

VLS-1 attitude model

The VLS-1 attitude model considered in this work is based on the 1st-stage of the rocket as described in Ramos (2011). It is defined in the mks unit system (meter, kilogram, second), with angles given in radians. However, all the results presented in this work are shown in degrees due to conversions from radians.

The attitude angles, roll (ϕ), pitch (θ), yaw (ψ), in the body coordinate system, X_b , Y_b , and Z_b , respectively aligned Up-East-South, have positive angular rotations around these axes by considering right-handed rotations, as depicted in the diagram of the VLS-1 shown in Fig. 2. The 1st-stage used four strap-on solid-fuel boosters with thrust vector control to steer the vehicle along the guidance path.



Source: Adapted from <https://en.wikipedia.org/wiki/File:VLS-1.svg>.

Figure 2. VLS-1 body axes and Euler angles.

The continuous-time state-space realization for the VLS-1 attitude model is as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{A}_c \mathbf{x}(t) + \mathbf{B}_c f_{sar}[\mathbf{u}(t - t_d)] + \mathbf{G}_c \mathbf{w}(t), \quad (1)$$

$$\mathbf{y}(t) = \mathbf{C} \mathbf{x}(t) + \mathbf{v}(t), \quad (2)$$

$$f_{sar}[\mathbf{u}(t)] = \begin{cases} \mathbf{u}_{\max} & \text{if } \mathbf{u}(t) \geq \mathbf{u}_{\max}, \\ \mathbf{u}(t) & \text{if } \mathbf{u}_{\min} < \mathbf{u}(t) < \mathbf{u}_{\max}, \\ \mathbf{u}_{\min} & \text{if } \mathbf{u}(t) \leq \mathbf{u}_{\min}. \end{cases} \quad (3)$$

The state vector $\mathbf{x}(t) \in \mathbb{R}^{16}$ is comprised of the following state variables:

$$\mathbf{x}^T(t) = [w \quad q \quad \theta \quad \theta_{b11} \quad \theta_{b12} \quad \theta_{b21} \quad \theta_{b22} \quad v \quad r \quad \psi \quad \psi_{b11} \quad \psi_{b12} \quad \psi_{b21} \quad \psi_{b22} \quad p \quad \phi]. \quad (4)$$

where v , and w , are the linear velocity components relative to Y_b and Z_b ; p , q , and r , are the angular velocities relative to rotations about X_b , Y_b , and Z_b ; θ_{b11} , and θ_{b21} , are the 1st and 2nd bending modes of $\theta(t)$; θ_{b12} , and θ_{b22} , are the time derivatives of the 1st and 2nd bending modes of $\theta(t)$; ψ_{b11} , and ψ_{b21} , are the 1st and 2nd bending modes of $\psi(t)$; ψ_{b12} , and ψ_{b22} , are the time derivatives of the 1st and 2nd bending modes of $\psi(t)$.

The four nozzles of the 1st-stage operate together as described in Palmerio (2017). Effectively, their movement can be addressed to three virtual nozzle angular deflections, in the input vector $\mathbf{u}^T(t) = [\beta_\theta \quad \beta_\psi \quad \beta_\phi]$, given in radians. Positive nozzle deflections in β_θ and β_ϕ generate positive rotations in θ and ϕ , while β_ψ operates inversely on ψ . The control actuators have limited nozzle deflections of $\pm 4^\circ$, represented by $\mathbf{u}_{\min}$ and $\mathbf{u}_{\max}$, and have a transport time delay t_d to simulate non-collocated sensor-actuator dynamics and other processing and communication delays.

The disturbance vector $\mathbf{w}^T(t) = [w_\theta \quad w_\psi]$ is an input vector, related to wind disturbances given in meters per second, affecting predominantly the pitch and yaw. The sensor-measured output vector is $\mathbf{y}^T(t) = [\theta \quad \psi \quad \phi]$, given in radians, and the sensor noise vector is $\mathbf{v}^T(t) = [v_\theta \quad v_\psi \quad v_\phi]$, simulated as Gaussian noises to assume the worst possible scenario where the whole measurable disturbance spectrum is excited with the same power.

The system is assumed to be time-invariant, and its matrices, $\mathbf{A}_c$, $\mathbf{B}_c$, $\mathbf{G}_c$, and $\mathbf{C}$, are parameterized at Max Q as shown in Ramos (2011), but with the bending modes enabled by modifying the following elements of the $\mathbf{A}_c$ matrix: $a_{3,4}$, $a_{3,6}$, $a_{10,11}$, and $a_{10,13}$, changing their values from 0 to 1.

To design and analyze the digital attitude controllers, it was used the zero-order hold equivalent state-space realization in the discrete time domain k , as follows:

$$\mathbf{x}(k) = \mathbf{A} \mathbf{x}(k-1) + \mathbf{B} f_{sar}[\mathbf{u}(k-d)] + \mathbf{G} \mathbf{w}(k-1), \quad (5)$$

$$\mathbf{y}(k) = \mathbf{C} \mathbf{x}(k) + \mathbf{v}(k). \quad (6)$$

The total discrete time delay is the positive integer $d = 1 + \lceil t_d/T_s \rceil$, with the ceiling operator $\lceil \cdot \rceil$ used to round up to an integer in the case of a fractional relation between t_d and the sampling time T_s .

The sampling time for the digital attitude controllers in this work must safely incorporate the 2nd bending mode frequency at 80.4 rad/s, or $f_b = 12.796$ Hz.

The rule of thumb shown in Stevens *et al.* (2016, p. 601), for aircraft control system design, is $T_s \leq 1/(4f_h)$. Since the VLS-1 is more critical due to its natural open-loop instability, an even safer rule was considered: $T_s \leq 1/(10f_h)$. Thus, a $T_s = 5$ milliseconds was selected.

In Fig. 1, the magnitude plots of the principal gains for pitch, yaw, and roll of the continuous- and discrete-time models of the VLS-1 are shown. It is possible to observe that both resonant peaks of the bending modes are well represented by the discrete-time equivalent model to be used for control system design and simulation.

The discrete-time model obtained using a sampling time of $T_s = 5$ milliseconds can represent the continuous system's principal gains for the whole sampled spectrum.

The implementation code for MATLAB used in this work can be downloaded from the MathWorks File Exchange website in Silveira (2025).

Functional approximation of the VLS-1 PI-D attitude controllers

VLS-1 PI-D control structure

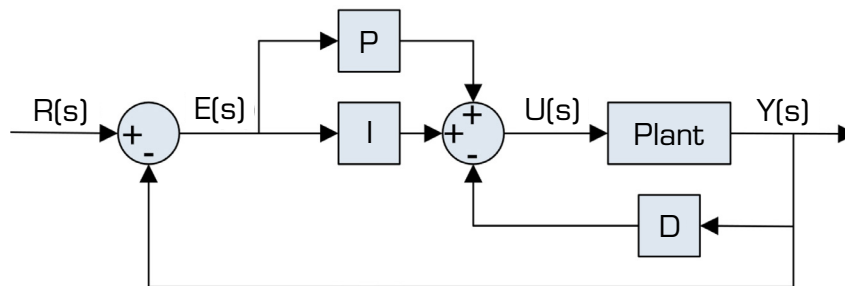
The PI-D controller is as follows:

$$U(s) = \left(k_p + k_i \frac{1}{s}\right) R(s) - \left(k_p + k_i \frac{1}{s} + k_d s\right) Y(s), \quad (7)$$

with k_p , k_i , and k_d being the proportional, integral, and derivative gains, respectively, and $U(s)$, $R(s)$, and $Y(s)$ being the control, the reference, and the controlled output signals, respectively. This is a two-degrees-of-freedom (2-DOF) control structure that uses PI actions to deal with the reference tracking and PID actions to deal with the output regulation. Such a control structure is depicted in the block diagram shown in Fig. 3 and can be generalized as follows:

$$U(s) = \frac{H(s)}{\Delta(s)} R(s) - \frac{F(s)}{\Delta(s)} Y(s). \quad (8)$$

This generalization is used later on to highlight an important remark regarding the improper use of open-loop-based relative stability analysis methods for 2-DOF control systems like the one shown in Eq. 8.



Source: Elaborated by the authors.

Figure 3. Block diagram of the PI-D control structure.

Different from the conventional PID topology, in PI-D the derivative part is not directly affected by the reference $R(s)$, avoiding the “derivative kick” when a step-like reference change occurs. Even so, noise and load disturbances fed back through the derivative path can still be amplified by the derivative action, generating control signal “chattering” (i.e., a noisy control signal), leading to undesirable outcomes, such as degradation of actuators, loss of fuel or power by the actuators, and excitation of oscillatory modes like the bending modes of the VLS-1 shown in Fig. 1.

The digital PI-D used in this work was obtained using Euler's backward approximation, leading to the following digital PI-D synthesis defined in the discrete-time shift operator domain q (i.e., $q^i M(q) := m(k+i)$, for any general $m(k)$ input or output signal):

$$U(q) = \frac{H(q)}{\Delta(q)} R(q) - \frac{F(q)}{\Delta(q)} Y(q), \quad (9)$$

With polynomials given by:

$$\Delta(q) = 1 - q^{-1}, \quad (10)$$

$$H(q) = (k_p + k_i T_s) + (-k_p) q^{-1} = h_0 + h_1 q^{-1}, \quad (11)$$

$$F(q) = \left(k_p + k_i T_s + \frac{k_d}{T_s}\right) + \left(-k_p - 2\frac{k_d}{T_s}\right) q^{-1} + \left(\frac{k_d}{T_s}\right) q^{-2} = f_0 + f_1 q^{-1} + f_2 q^{-2}, \quad (12)$$

and the difference equation to implement the functional digital approximation is given by:

$$u(k) = u(k-1) + h_0 r(k) + h_1 r(k-1) - f_0 y(k) - f_1 y(k-1) - f_2 y(k-2). \quad (13)$$

It is important to remark that for systems with null initial conditions and/or stable systems, the shift operator q is equivalent to the discrete frequency domain $q := z = e^{j\omega T_s}$, in which ω is the continuous frequency domain (Astrom and Wittenmark 2011).

VLS-1 PI-D tuning

The VLS-1 PI-D tuning and performance were based on the linear quadratic regulator (LQR) method described in Silva *et al.* (2019), and using the parameters in Ramos (2011), for the condition in which the VLS-1 is under maximum aerodynamic pressure:

$$\frac{\theta(s)}{\beta_\theta(s)} = \frac{-\bar{M}_{\beta_z}}{s^2 - \bar{M}_\alpha} = \frac{-(-7.3734)}{s^2 - (4.077)}, \quad (14)$$

$$\frac{\psi(s)}{\beta_\psi(s)} = \frac{\bar{N}_{\beta_y}}{s^2 - \bar{N}_\beta} = \frac{(-7.5648)}{s^2 - (4.0726)}, \quad (15)$$

$$\frac{\phi(s)}{\beta_\phi(s)} = \frac{\bar{L}_{\beta_x}}{s^2 + \bar{L}_p s} = \frac{(61.0492)}{s^2 + (0.098859)s}. \quad (16)$$

These simplified attitude models can be generalized into a canonical 2nd-order system of the form:

$$\frac{Y(s)}{U(s)} = \frac{k_s \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}, \quad (17)$$

with k_s , ω_n , and ζ , being the static gain, the natural frequency, and the damping factor, respectively.

Converting this system into a state-space realization using the state vector $[y(t) \quad \dot{y}(t)]^T$, leads to:

$$\begin{bmatrix} \dot{y}(t) \\ \ddot{y}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} y(t) \\ \dot{y}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ k_s\omega_n^2 \end{bmatrix} u(t). \quad (18)$$

The PI-D is now correlated with the system basis, as the control law is rewritten in the time domain as follows:

$$u(t) = k_p [r(t) - y(t)] + k_i \int [r(t) - y(t)] dt - k_d \dot{y}(t), \quad (19)$$

such that the controller's integral term is assumed to be a new state variable

$$e_i(t) = \int [r(t) - y(t)] dt, \quad (20)$$

leading to the following augmented state-space realization:

$$\begin{bmatrix} \dot{y}(t) \\ \ddot{y}(t) \\ \dot{e}_i(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\omega_n^2 & -2\zeta\omega_n & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} y(t) \\ \dot{y}(t) \\ e_i(t) \end{bmatrix} + \begin{bmatrix} 0 \\ k_s\omega_n^2 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(t). \quad (21)$$

The regulatory control problem assumes $r(t) = 0$, $\forall t$, and the state-feedback control law is

$$u(t) = -\mathbf{K} \begin{bmatrix} y(t) & \dot{y}(t) & e_i(t) \end{bmatrix}^T, \quad (22)$$

relating the LQR gain and the PI-D gains as follows:

$$\mathbf{K} = \begin{bmatrix} k_p & k_d & -k_i \end{bmatrix}. \quad (23)$$

By using the LQR optimization weighting matrices as specified by Silva *et al.* (2019), with $\mathbf{Q} = \text{diag}(0.1 \ 1 \ 0.2)$ and $\mathbf{R} = 0.4$, in which the solution to the LQR problem is standardized in several computational toolboxes as $\mathbf{K} = \text{lqr}(\mathbf{A}_c, \mathbf{B}_c, \mathbf{Q}, \mathbf{R})$, the following PI-D gains for the pitch, yaw, and roll attitude controllers were obtained:

$$\begin{aligned} \mathbf{K}_\theta &= \begin{bmatrix} 2.3009 & 1.7675 & -0.7071 \end{bmatrix}, \\ \mathbf{K}_\psi &= \begin{bmatrix} -2.2793 & -1.7614 & 0.7071 \end{bmatrix}, \\ \mathbf{K}_\phi &= \begin{bmatrix} 1.5840 & 1.5958 & -0.7071 \end{bmatrix}. \end{aligned} \quad (24)$$

Predictive minimum variance control (PMVC) with full state-feedback

The proposition of a PMVC with full state-feedback assumes a predicted state-feedback control law of the form:

$$\Delta \mathbf{u}(k) = \mathbf{K} [\mathbf{r}(k) - \hat{\mathbf{x}}_a(k + N_x)], \quad (25)$$

in which $\Delta \mathbf{u}(k) = \mathbf{u}(k) - \mathbf{u}(k-1)$, $\mathbf{\Lambda}$ is a diagonal matrix for control effort weighting, $\mathbf{K}$ is a state-feedback gain matrix, $\mathbf{r}(k)$ is a vector of state-reference signals, and $\hat{\mathbf{x}}_a(k + N_x)$ is the prediction of N_x steps in the future of the augmented state vector to add discrete integrators to the system inputs:

$$\mathbf{x}_a^T(k) = \begin{bmatrix} \mathbf{y}(k) & \Delta \mathbf{x}(k) \end{bmatrix}. \quad (26)$$

Then, the PMVC optimization problem with full state-feedback is reformulated as follows:

$$\min_{\Delta \mathbf{u}(k)} J = \mathbf{E} \left[\mathbf{g}^T(k + N_x) \mathbf{g}(k + N_x) \right], \quad (27)$$

$$\mathbf{g}(k + N_x) = \mathbf{\Lambda} \Delta \mathbf{u}(k) + \mathbf{K} \hat{\mathbf{x}}_a(k + N_x) - \mathbf{K} \mathbf{r}(k), \quad (28)$$

subject to the augmented system realization with new matrices $\mathbf{A}_a$, $\mathbf{B}_a$, $\mathbf{G}_a$, and $\mathbf{C}_a$:

$$\begin{bmatrix} \mathbf{y}(k) \\ \Delta \mathbf{x}(k) \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{C} \mathbf{A} \\ \mathbf{0} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{y}(k-1) \\ \Delta \mathbf{x}(k-1) \end{bmatrix} + \begin{bmatrix} \mathbf{C} \mathbf{B} \\ \mathbf{B} \end{bmatrix} \Delta \mathbf{u}(k-d) + \begin{bmatrix} \mathbf{I} & \mathbf{C} \mathbf{G} \\ \mathbf{0} & \mathbf{G} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{v}(k) \\ \Delta \mathbf{w}(k-1) \end{bmatrix}, \quad (29)$$

$$\mathbf{y}_a(k) = \begin{bmatrix} \mathbf{C} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{y}(k) \\ \Delta \mathbf{x}(k) \end{bmatrix}.$$

The solution to the problem in Eq. 27 requires the PMVC state predictor (Silveira *et al.* 2020):

$$\begin{aligned} \hat{\mathbf{x}}_a(k + N_x) &= \left(\mathbf{A}_a^{N_x} - \mathbf{A}_a^{(N_x-1)} \mathbf{L} \mathbf{C}_a \right) \bar{\mathbf{x}}_a(k) + \mathbf{B}_a \Delta \mathbf{u}(k) \\ &+ \begin{bmatrix} \mathbf{A}_a^1 \mathbf{B}_a \\ \vdots \\ \mathbf{A}_a^{(N_x-1)} \mathbf{B}_a \end{bmatrix}^T \underbrace{\begin{bmatrix} \Delta \mathbf{u}(k-1) \\ \vdots \\ \Delta \mathbf{u}(k - N_x + 1) \end{bmatrix}}_{\Delta \mathbf{u}(k-1)} + \mathbf{A}_a^{(N_y-1)} \mathbf{L} \mathbf{y}(k), \end{aligned} \quad (30)$$

$$\bar{\mathbf{x}}_a(k) = (\mathbf{A}_a - \mathbf{L} \mathbf{C}_a) \bar{\mathbf{x}}_a(k-1) + \mathbf{B}_a \Delta \mathbf{u}(k - N_x) + \mathbf{L} \mathbf{y}(k-1). \quad (31)$$

The matrices $\mathbf{K}$ and $\mathbf{L}$ are the LQR and Kalman filter gains of an LQG design, used to “optimally detune” the predictive minimum variance problem.

By solving the optimization problem in Eq. 27, the optimal control increment is as follows:

$$\Delta \mathbf{u}(k) = (\mathbf{K} \mathbf{B}_a + \mathbf{\Lambda})^{-1} \left[\mathbf{K} \mathbf{r}(k) - \mathbf{M}_x \bar{\mathbf{x}}_a(k) - \mathbf{M}_u \Delta \mathbf{u}(k-1) - \mathbf{M}_y \mathbf{y}(k) \right], \quad (32)$$

$$\begin{aligned}
\mathbf{M}_x &= \mathbf{K} \left(\mathbf{A}_a^{N_x} - \mathbf{A}_a^{(N_x-1)} \mathbf{L} \mathbf{C}_a \right), \\
\mathbf{M}_u &= \mathbf{K} \begin{bmatrix} \mathbf{A}_a^1 \mathbf{B}_a & \cdots & \mathbf{A}_a^{(N_x-1)} \mathbf{B}_a \end{bmatrix}, \\
\mathbf{M}_y &= \mathbf{K} \mathbf{A}_a^{(N_x-1)} \mathbf{L},
\end{aligned} \tag{33}$$

and the incremental control law is given by: $\mathbf{u}(k) = \mathbf{u}(k-1) + \Delta \mathbf{u}(k)$.

Relative stability and robustness analysis

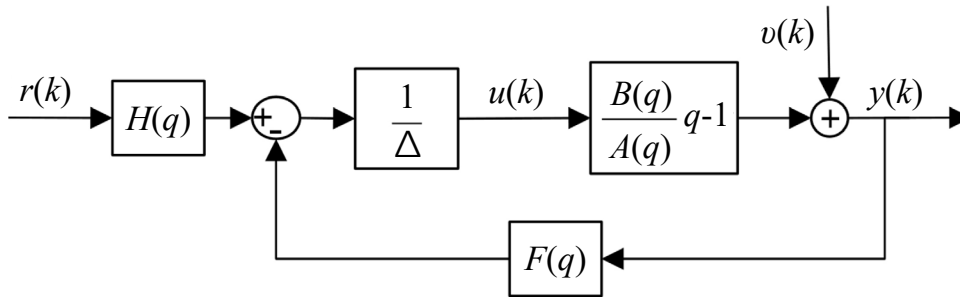
The LQR method can guarantee the asymptotic closed-loop stability for both the PI-D controllers and the PMVC. However, for a safe and reliable implementation based on linear control methods, the relative stability analysis must be investigated to guarantee robustness to model uncertainties and disturbance rejection. Thus, in this section, important remarks are covered to adequately address control-loop robustness through gain margin and phase margin analysis when dealing with unstable plants such as the VLS-1.

The discussion begins by covering single-input single-output (SISO) transfer function cases with 2-DOF control structures, like the PI-D, and then the more general multivariate (MIMO) state-space case for the LQG/LTR-based PMVC, showing the inappropriateness of open-loop analysis for unstable systems and a workaround method using stable closed-loop sensitivity and cosensitivity functions.

SISO transfer function case

The PID-based digital syntheses can be generalized in a reference-signal-tracking control structure as shown in Fig. 4, defined in this work as

$$\Delta(q)u(k) = H(q)r(k) - F(q)y(k). \tag{34}$$



Source: Elaborated by the authors.

Figure 4. Reference-signal-tracking problem generalization.

$\Delta(q) = \Delta = 1 - q^{-1}$ is the discrete-difference operator, so the control difference is $\Delta u(k) = u(k) - u(k-1)$, and $H(q)$ and $F(q)$ are polynomials like those from the functional PI-D approximation in Eqs. 11 and 12.

The controlled plant is assumed to be $A(q)y(k) = B(q)q^{-d}u(k)$, and if it is open-loop stable, it is common to assess the control-loop stability and robustness based on the open-loop system by opening the feedback loop shown in Fig. 4, and the following direct-loop system is considered:

$$D_L(q) = \frac{y(k)}{r(k)} = H(q) \frac{1}{\Delta} \frac{B(q)}{A(q)} q^{-d}. \tag{35}$$

Assuming null initial conditions, $q := z$, the z-transform can be applied, converting this system to the discrete frequency domain $z = e^{j\omega T_s}$. Thus, Bode's stability criterion can be used to obtain gain and phase margins based on $D_L(z)$ gain and phase plots. Note, however, that Eq. 35 does not explicitly consider the effects of the output disturbance $v(k)$ in the analysis. More importantly, the analysis aiming to obtain gain and phase margins for open-loop unstable plants is not possible, and the following remarks are given.

- Remark 1 – In Eq. 35, if the open-loop transfer function $D_L(z)$ has unstable poles, then Bode's stability criterion cannot be applied to obtain gain and phase margins, since an unstable $D_L(e^{j\omega T_s})$ does not respond sinusoidally.
- Remark 2 – The same theoretical concept of Remark 1 applies to unstable closed-loop systems, as their frequency response is not sinusoidal. Thus, the relative stability analysis by gain and phase margins via sensitivity functions can only be made if the closed-loop system is stable.
- Remark 3 – In Eq. 35, no feedback filter can participate in the analysis, such as the $F(q)$ filter, since the direct-loop path from $r(k)$ to $y(k)$ is under consideration without the feedback signal. Thus, 2-DOF PID algorithms, such as the PI-D, require closed-loop analysis for proper frequency response evaluation.

An example of the violation of such remarks is the reproduction of the gain and phase margin analysis as conducted in the work of Silva *et al.* (2019), depicted in Fig. 5, where the authors used the unstable direct loop transfer function

$$D_L(s) = \left(\frac{k_p s + k_i}{s} \right) \left[\frac{\theta(s)}{\beta_\theta(s)} \right] (k_d s + 1), \quad (36)$$

in which $\theta(s)/\beta_\theta(s)$ is unstable, and $k_d s + 1$ is part of the feedback loop and should not be in the open-loop path. In Fig. 5, even though the presented Bode diagram is correct, it was misinterpreted by Silva *et al.* (2019) in terms of robustness, since negative gain margins were assumed to be adequate.

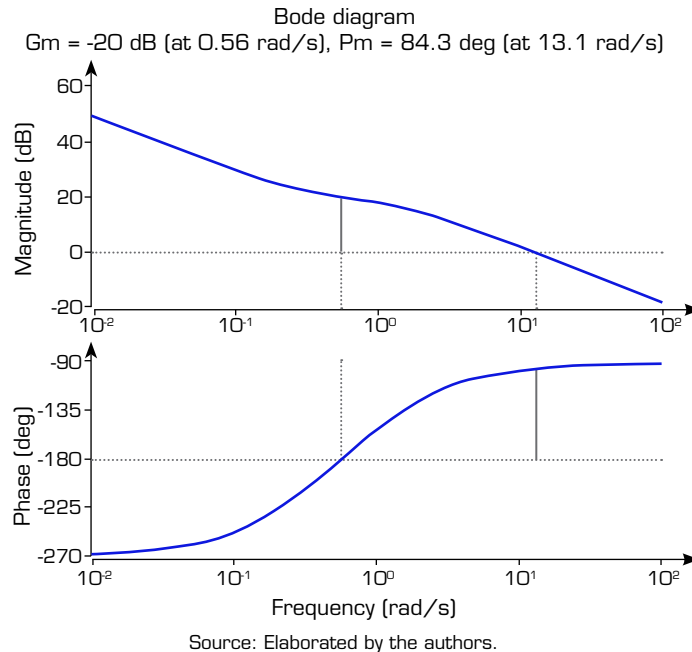


Figure 5. Gain and phase margins based on the Bode plot of an unstable direct loop. Reproduction based on the controller and plant parameters in Silva *et al.* (2019).

The robustness analysis method investigated in this work considers the stable closed-loop system from the perspective of the system's sensitivity from the reference $r(k)$ to $y(k)$, and from the output disturbance $v(k)$ to $y(k)$, as follows:

$$y(k) = \frac{H(q)B(q)q^{-d}}{\Delta A(q) + F(q)B(q)q^{-d}}r(k) + \frac{\Delta A(q)}{\Delta A(q) + F(q)B(q)q^{-d}}v(k). \quad (37)$$

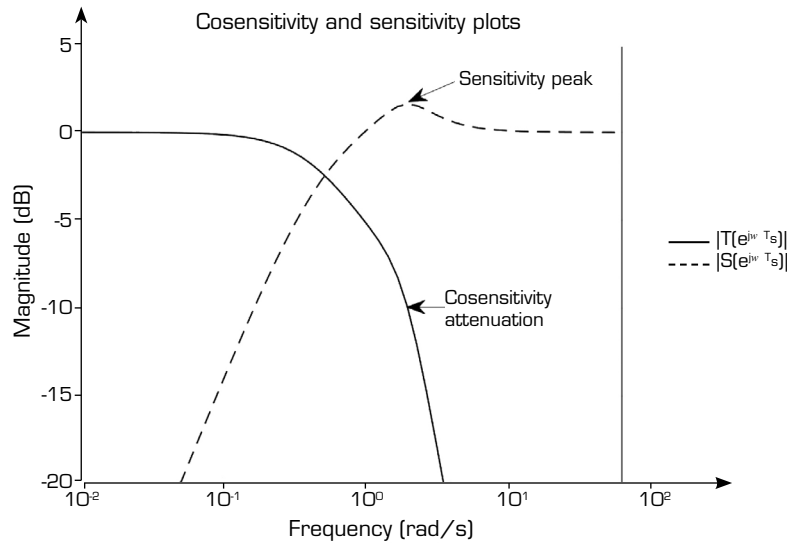
Due to null initial conditions, these transfer functions can be rewritten in the discrete frequency domain $z = e^{j\omega T_s}$, so that their z -transform leads to

$$Y(z) = T(z)R(z) + S(z)V(z), \quad (38)$$

where $S(z)$ is the sensitivity function and $T(z)$ is the complementary sensitivity (cosensitivity) function, in which the overall system sensitivity in terms of the closed-loop amplitude spectrum is given by:

$$|T(e^{j\omega T_s})| + |S(e^{j\omega T_s})| = 1. \quad (39)$$

For a closed-loop control system to have guaranteed reference-signal-tracking robustness, the control system must exhibit considerable magnitude attenuation along the frequencies where the system's sensitivity function $|S(e^{j\omega T_s})|$ is high, as can be seen in the example in Fig. 6.



Source: Elaborated by the authors.

Figure 6. Robust closed-loop control example: strong attenuation of the sensitivity peak.

The more the sensitivity is attenuated, the more the relative stability margins and robustness increase. The magnitude plots of the sensitivity functions, when analyzed together, can be used to quantify the relative stability by gain and phase margins using the following results from Nyquist's stability criterion, as described in Seborg *et al.* (2003):

$$\begin{aligned} G_{M_{dB}} &\geq \min \left[20 \log_{10} \left(\frac{m_s}{m_s - 1} \right), 20 \log_{10} \left(1 + \frac{1}{m_t} \right) \right], \\ P_{M_{deg}} &\geq \left(\frac{180^\circ}{\pi} \right) \min \left\{ \left[2 \sin^{-1} \left(\frac{1}{2m_s} \right) \right], \left[2 \sin^{-1} \left(\frac{1}{2m_t} \right) \right] \right\}, \end{aligned} \quad (40)$$

with $G_{M_{dB}}$ in decibels and $P_{M_{deg}}$ in degrees, and m_t and m_s being the linear peak gains of the cosensitivity and sensitivity functions:

$$\begin{aligned} m_t &= \max_{\omega} |T(e^{j\omega T_s})|, \\ m_s &= \max_{\omega} |S(e^{j\omega T_s})|. \end{aligned} \quad (41)$$

The min operator in Eq. 40 is not explicitly considered in Seborg *et al.* (2003). However, the min operator must be adopted to ensure that the control system designer does not neglect the worst calculated margins.

Multiple-input multiple-output (MIMO) state-space case

The sensitivity function analysis for MIMO systems was addressed in Doyle and Stein (1979) after the seminal paper by Doyle (1978) on the “Guaranteed margins for LQG regulators,” in which the abstract read simply: “There are none.” These seminal papers were the preparatory steps for the proposition of the LQG/LTR design based on the principal gains of MIMO systems (Doyle and Stein 1981).

In the multivariate case, the increased number of input-output pairs elevated the complexity of analyzing gain and phase plots in the frequency domain; thus, the sensitivity and cosensitivity analyses by means of principal gains (singular values) became an important method for frequency response analysis in control system design for aerospace vehicles (Stevens *et al.* 2016). The technique is similar to the classical SISO case, changing slightly to incorporate the maximum among all sensitivity peaks, as follows:

$$\begin{aligned} m_t &= \sup_{\omega} |\mathbf{T}(e^{j\omega T_s})|, \\ m_s &= \sup_{\omega} |\mathbf{S}(e^{j\omega T_s})|, \end{aligned} \quad (42)$$

where the supremum operator is considered, since the number of magnitude plots is equivalent to the number of input-output pairs. Similarly, the relation between the sensitivity and cosensitivity is rewritten in matrix form:

$$|\mathbf{T}(e^{j\omega T_s})| + |\mathbf{S}(e^{j\omega T_s})| = \mathbf{I}, \quad (43)$$

with $\mathbf{I}$ being an identity matrix with its dimension equivalent to the number of input-output pairs.

The separation principle, concerning the independent stability analysis of the state estimator and the state-feedback controller, is no longer considered. Thus, the LQG/LTR as a complete dynamic compensator comprised of an optimal state estimator and a full-estimated-state-feedback controller is given by:

$$\Delta \mathbf{u}(k) = \mathbf{K} [\mathbf{r}(k) - \bar{\mathbf{x}}_a(k)] = \mathbf{K}_s \mathbf{r}_s(k) - \mathbf{K} \bar{\mathbf{x}}_a(k), \quad (44)$$

with $\mathbf{K}_s$ and $\mathbf{r}_s(k)$ being subsets of elements of $\mathbf{K}$ and $\mathbf{r}(k)$, respectively. These subsets are related to the number of input-output pairs of the MIMO system, in which the servo tracking problem is necessary. For example, for the VLS-1, these are the first three reference signals in $\mathbf{r}(k)$, since all the others are null, to regulate the discrete difference of the state variables, i.e., $\Delta \bar{\mathbf{x}}(k)$, as shown in the augmented state model in Eq. 29. Substituting this control law into the controlled system (assuming $d=1$, for simplification), leads to

$$\mathbf{x}_a(k+1) = \mathbf{A}_a \mathbf{x}_a(k) - \mathbf{B}_a \mathbf{K} \bar{\mathbf{x}}_a(k) + \mathbf{B}_a \mathbf{K}_s \mathbf{r}_s(k), \quad (45)$$

and the state estimator in closed-loop is

$$\bar{\mathbf{x}}_a(k+1) = (\mathbf{A}_a - \mathbf{B}_a \mathbf{K} - \mathbf{L} \mathbf{C}_a) \bar{\mathbf{x}}_a(k) + \mathbf{L} \mathbf{C}_a \mathbf{x}_a(k) + \mathbf{B}_a \mathbf{K}_s \mathbf{r}_s(k). \quad (46)$$

The complete dynamic compensator for the joint analysis is then defined as:

$$\underbrace{\begin{bmatrix} \mathbf{x}_a(k+1) \\ \bar{\mathbf{x}}_a(k+1) \end{bmatrix}}_{\mathbf{x}_d(k+1)} = \underbrace{\begin{bmatrix} \mathbf{A}_a & -\mathbf{B}_a \mathbf{K} \\ \mathbf{L} \mathbf{C}_a & \mathbf{A}_a - \mathbf{B}_a \mathbf{K} - \mathbf{L} \mathbf{C}_a \end{bmatrix}}_{\mathbf{A}_d} \underbrace{\begin{bmatrix} \mathbf{x}_a(k) \\ \bar{\mathbf{x}}_a(k) \end{bmatrix}}_{\mathbf{x}_d(k)} + \underbrace{\begin{bmatrix} \mathbf{B}_a \mathbf{K}_s \\ \mathbf{B}_a \mathbf{K}_s \end{bmatrix}}_{\mathbf{B}_d} \mathbf{r}_s(k), \quad (47)$$

$$\mathbf{y}(k) = \underbrace{\begin{bmatrix} \mathbf{C}_a & \mathbf{0} \end{bmatrix}}_{\mathbf{C}_d} \mathbf{x}_d(k). \quad (48)$$

The cosensitivity system can be obtained by converting the system to the matrix transfer function

$$\mathbf{T}(z) = \mathbf{C}_d (z\mathbf{I} - \mathbf{A}_d)^{-1} \mathbf{B}_d, \quad (49)$$

and the sensitivity matrix transfer function

$$\mathbf{S}(z) = \mathbf{I} - \mathbf{T}(z). \quad (50)$$

It is important to highlight that the complete dynamic compensator derived in this work is matched to the state-space canonical representation developed for the PMVC case, and differences in the way the state estimator is implemented in the MIMO case can generate different dynamic compensators, as shown in Stevens *et al.* (2016) for continuous-time systems.

RESULTS AND DISCUSSION

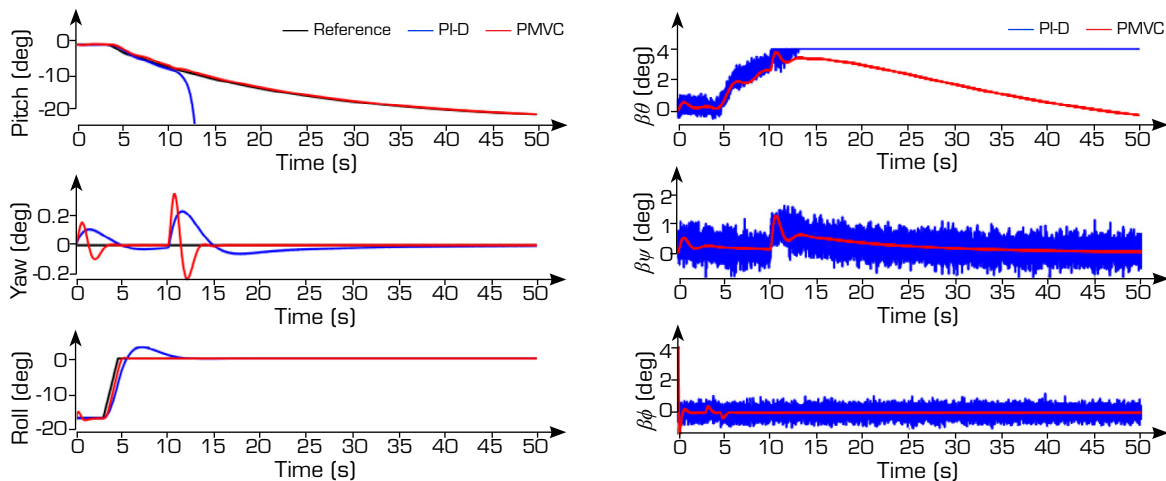
In this section, a 50-second simulation of the VLS-1 is presented while executing pitch-over and roll program maneuvers based on the guidance data shown in Yamada *et al.* (2024), with wind disturbances of 6 m/s at liftoff and 20 m/s 10 seconds later. Note, however, that this simulation is not necessarily intended to accurately reconstruct the VLS-1 V02 failure, including its timing. Throughout the 50-second simulation, the model remains time-invariant, and its parameters reflect the flight condition at Max Q. Therefore, the simulations cannot be correlated second-by-second with the actual atmospheric flight of the VLS-1 V02 shown in the video footage presented by Retro Space HD (2022). Nevertheless, the simulation model may be regarded as a functional approximation of the launch vehicle between approximately 40 and 50 seconds after liftoff, corresponding to the flight condition at which the model was linearized by Ramos (2011).



Since the adopted simulation model is time-invariant, an adaptive control implementation for the PI-D and PMVC controllers is unnecessary. It is important to emphasize, however, that the functional approximation of the PI-D controller was optimized specifically for this operating condition and model. When vibrations, wind loads, delays, and nozzle deflection limits are introduced, the original controller design indeed leads to control failure, particularly during the execution of combined maneuvers prescribed by the guidance data. This guidance profile was specifically employed for the sequential assessment of a broad range of attitude references reflecting near-limit nozzle deflections experienced by the VLS during the mitigation of 20 m/s wind loads under the intense vibratory regime at Max Q.

For the sake of compactness, only a single and standardized tuning of the PMVC was used: $N_x = 11$, $\Lambda = \mathbf{I}_3$, LQR $\mathbf{Q} = \mathbf{C}_a^T \mathbf{C}_a$ and $\mathbf{R} = 10^3 \mathbf{I}_3$, Kalman filter $\mathbf{Q} = \mathbf{I}_9$, and $\mathbf{R} = \mathbf{I}_3$. Theoretically, with the defined state-prediction horizon N_x , the controller would guarantee the closed-loop stability up to $t_d = 50$ ms. Effectively, although not shown, the PMVC handled up to $t_d = 120$ ms with $N_x = 11$.

Figure 7 presents the results from simulations with vibrations being captured by the attitude sensors. The zero-mean Gaussian noises were generated with a linear power of 10^{-10} (rad/h)². This power is less than the noise power of angular sensor technologies, such as the interferometric fiber optic gyroscope, studied for the more recent and advanced Brazilian microsatellite launch vehicle, VLM-1, in which the theoretical Gaussian noise power is 10^{-5} (rad/h)² (Nunes *et al.* 2022). Even though these noises were barely visible in the graphics of the attitude angles, the control signals, on the other hand, showed excessive chattering, saturation, and eventual loss of control.



Source: Elaborated by the authors.

Figure 7. Attitude angles on the left graphics and control signals on the right. The vibrations simulated with noises' linear power of $\sigma_v^2 = 10^{-10}$ were barely visible, but their effect due to feedback in the PI-D was catastrophic, leading to high control signal chattering, saturation, and instability.

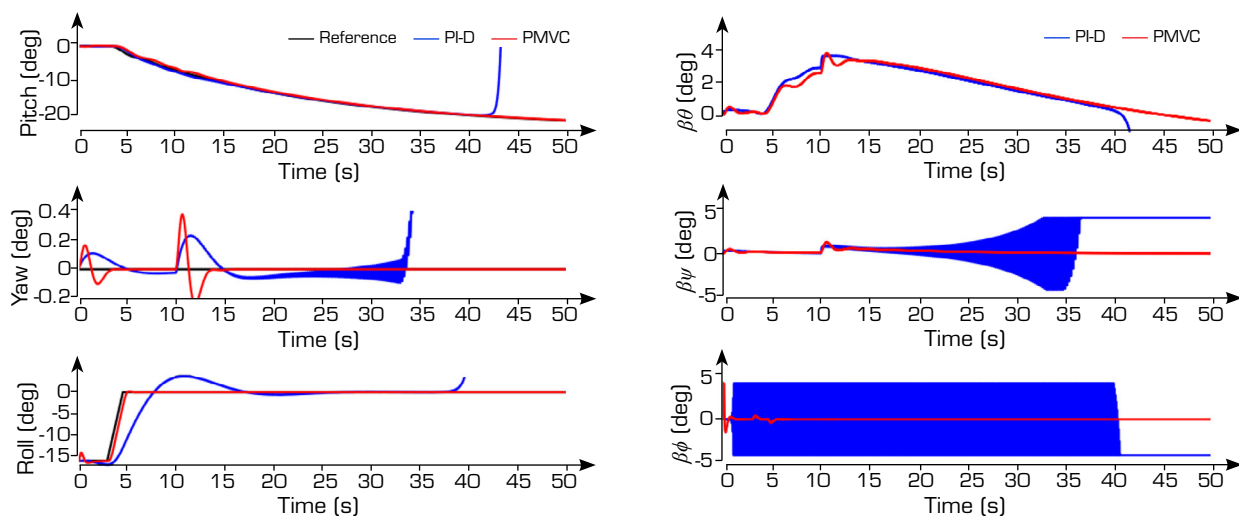
As observed in Fig. 7, instability in the PI-D case begins to develop shortly after the introduction of the 20 m/s wind disturbance at $t = 10$ s. Such a high-magnitude disturbance, combined with sensor noise, drives the nozzle to its saturation limit of 4° while also producing significantly higher control signal chattering when compared to the PMVC case. From a control theory perspective, this behavior characterizes a loss of closed-loop stability caused by a plausible and expected atmospheric disturbance, as reported by Yamada *et al.* (2024). One could argue, however, that since the wind disturbance is introduced at $t = 10$ s, i.e., before the transonic regime in a real flight scenario, the PI-D controller during this specific phase of operation might still have been capable of maintaining stability. Under this interpretation, after approximately $t = 30$ s, the control signals would likely remain sufficiently far from the saturation limits, thereby accommodating disturbances like those observed for the PMVC in the final instants of the simulation. Nevertheless, there is no guarantee that an equivalent wind disturbance would not occur later during the Max Q phase. Furthermore, another issue arises, one that lies beyond the strict scope of closed-loop stability analysis but still

within the thesis discussed by Palmerio (2017): are the observed levels of control signal chattering severe enough to compromise the structural integrity of the VLS-1 to the point of triggering an explosion during second-stage ignition? Although this question cannot be conclusively answered within the present work, the closed-loop stability degradation caused by wind disturbances and sensor noise remains, by itself, a significant concern.

In another simulation, the absence of vibrations and sensor noise is assumed to isolate and analyze the effects of digital signal processing delays and/or non-collocated actuator-sensor delays. Under such conditions, the rigid-body and flexible dynamics may interact, generating resonant oscillations that, if not properly suppressed, can ultimately lead to instability (Wie *et al.* 2012).

In Fig. 8, the effects of non-collocated sensor-actuator and time delays are shown. It was found that the PI-D would lose stability if $t_d \geq 10$ milliseconds. Figure 8 depicts the case with $t_d = 20$ ms, since the visual impact of this result is stronger in terms of the PI-D instability.

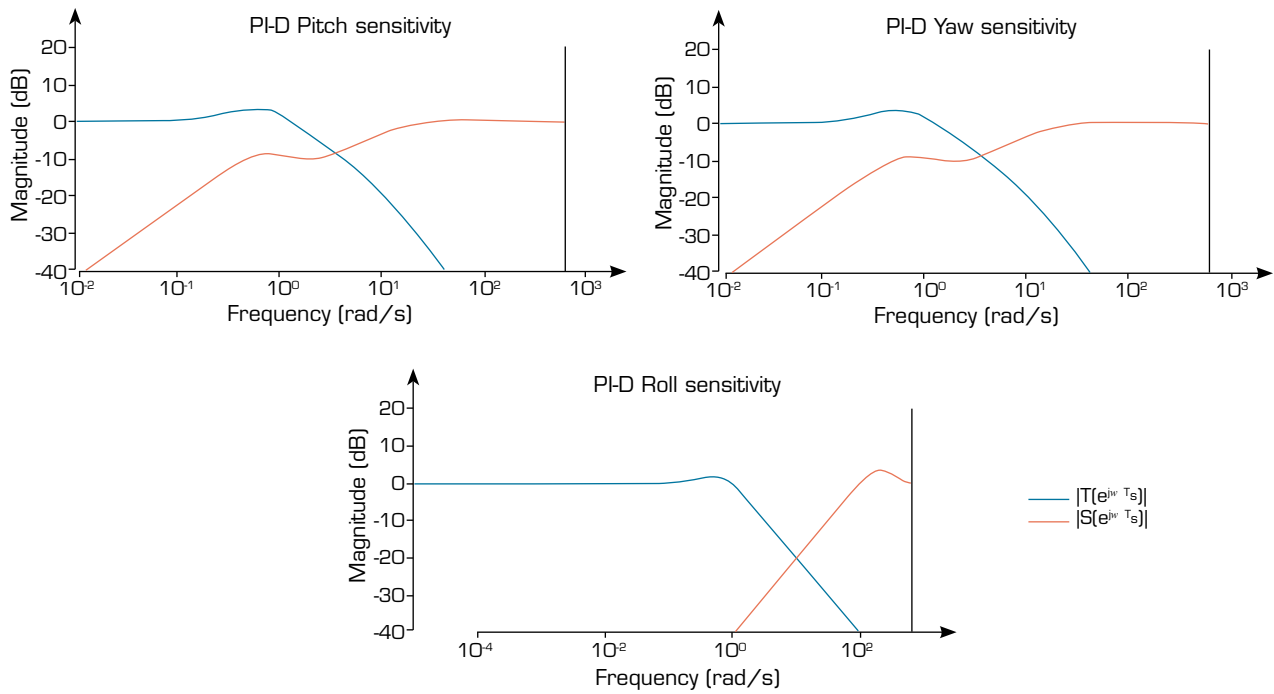
In the results shown in Fig. 8, the problems associated with the PI-D control loop are present from the very beginning of the simulation, being particularly evident in the roll control signal, which exhibits a “bang-bang” control behavior. It is important to remark, however, that notch filters are absent from this PI-D implementation. Nevertheless, Silva (2014) showed that even with the notch filters and with a -40 dB/dec loop-gain roll-off at frequencies above the control bandwidth – required to avoid destabilization due to unmodeled high-frequency dynamics and to attenuate high-frequency noise (Wie *et al.* 2012), as depicted in Fig. 9 – these measures were not enough to avoid loss of stability under the same time delay of $t_d \geq 2T_s$ investigated by Silva (2014, p. 176), since the provided attenuation is insufficient to suppress the first bending mode resonant peak previously shown in Fig. 1. Similarly, introducing delays in the simulation also degraded the stability of both the pitch and yaw control-loops, in which it is possible to observe that around $t = 33$ s the yaw control signal reached its physical limits as the resonant effect built up, and approximately at $t = 40$ s, both the pitch and roll loops became unstable.



Source: Elaborated by the authors.

Figure 8. Attitude angles on the left graphics and control signals on the right. Results for simulated non-collocated sensor-actuator and time delays. Instability with the PI-D was observed for $t_d \geq 10$ ms, but this case in the graphics was simulated using $t_d = 20$ ms to better depict the problem.

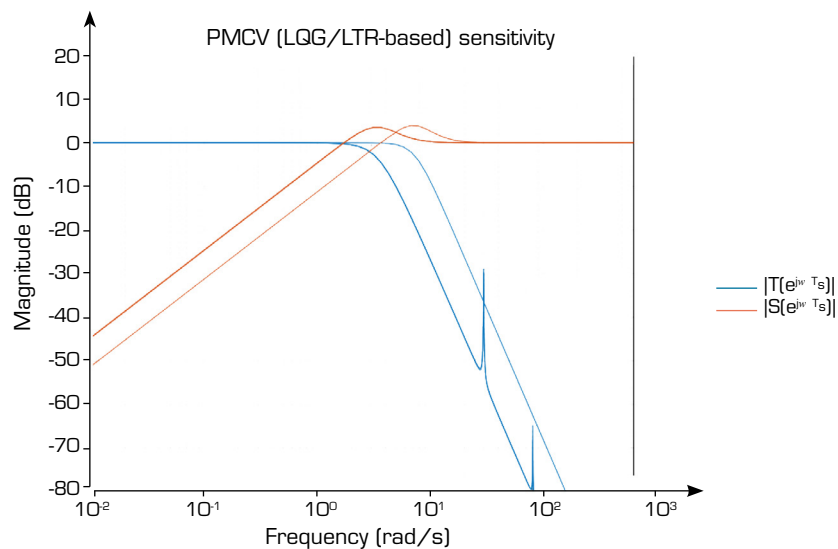
The sensitivity plots for the designed PI-D control loops are shown in Fig. 9. Since the designs were handled as three SISO systems, they are shown separately. For all three cases, the cosensitivity exhibited peaks beyond 0 dB, characterizing a resonant peak in the closed loop from the reference to the output. For the roll control case, this problem can be neglected because the sensitivity is extremely low where the cosensitivity exhibits its peak. However, for the pitch and yaw, the resonant peak works adversely to attenuate disturbances, thus diminishing the robustness to model uncertainties, vibrations, and sensor noise.



Source: Elaborated by the authors.

Figure 9. Sensitivity and cosensitivity plots of the PI-D controllers are analyzed based on simplified plant models of 2nd-order without the bending modes.

The relative stability analysis for the PMVC is shown in Fig. 10. In this case, the MIMO 16th-order model, assuming no delay ($d = 1$), was used to plot the principal gains, represented by the superior and inferior singular values of the multivariate sensitivity and cosensitivity systems. It is possible to observe the presence of the bending modes in the inferior cosensitivity plots.



Source: Elaborated by the authors.

Figure 10. Superior and inferior principal gains of the sensitivity and cosensitivity plots of the PMVC (LQG/LTR-based) were analyzed based on the 16th-order model with bending modes.

From a comparative viewpoint, the sensitivity analysis of the PMVC shown in Fig. 10 seems worse than the PI-D cases because the superior principal gain of the sensitivity is not attenuated by the superior principal gain of the cosensitivity, thus not exhibiting robustness to disturbances from a qualitative viewpoint when compared to the PI-D.

Despite not exhibiting qualitatively robust sensitivity characteristics compared to the PI-D, the PMVC was able to guarantee closed-loop stability where the PI-D had failed. This can be related to the more accurate model used in the design and the time-delay compensation handled by MPC, thus allowing the PMVC to work with less conservative robustness margins.

In Table 1 are shown the gain margins, phase margins, and sensitivity peaks for all designed controllers. From these results, it is possible to confirm that the PMVC has exhibited worst-case phase margins below those obtained by the PI-D, but a cosensitivity maximum peak of 1.0, thus without resonant peaks.

Table 1. Gain margins, phase margins. and sensitivity peaks.

Margins	PI-D Pitch	PI-D Yaw	PI-D Roll	PMVC
$G_{M_{dB}} \geq$	4.3988 dB	4.4123 dB	5.1055 dB	6.0206 dB
$P_{M_{deg}} \geq$	38.4989°	38.6550°	39.0306°	36.9616°
m_t	1.5166	1.5107	1.2500	1.0000
m_s	1.0599	1.0612	1.4967	1.5774

Source: Elaborated by the authors.

For flight control systems, the targeted robustness margins are commonly within the robust performance range, that is, a range of values to simultaneously address fast servo tracking and fast disturbance recovery while still guaranteeing elevated gain and phase margins to avoid loss of stability. In Stevens *et al.* (2016), the robust performance range is defined within 6 to 15 dB and 30° to 60°, in which only the PMVC was able to lie within.

CONCLUSION

In this work, the effects of vibrations and time delays acting on functional approximations of the VLS-1 control systems were investigated. The non-linear simulation model considered the attitude dynamics during the Max Q phase, with bending modes enabled, actuator saturation, and time delays. The main results were the difficulties the PI-D controller had when dealing with noises and delays that led to instability, and the successful management of such a complex scenario by the PMVC with full state feedback using LQG/LTR-based loop shaping.

The misuse of relative stability analysis by Bode's stability criterion when the controlled system is open-loop unstable, such as the VLS-1, was also investigated, and an alternative classical method based on sensitivity analysis and its use with modern control state-space methods to deal with MIMO systems was presented.

Using the sensitivity function analysis, it was possible to highlight the lack of robustness of the functional VLS-1 PI-D approximation, in terms of gain and phase margins to achieve the robust performance range for flight control systems, as shown by Stevens *et al.* (2016).

The PMVC simulated loop time was 0.23545 μ s, and the PI-D was 5.5674 μ s. These were the average times after processing 10001 iterations using MATLAB R2021a on an AMD Ryzen 5 5600 processor, with 32 GB of DDR4 RAM at 3200 MHz. Thus, both controllers are certainly fast enough, and onboard processing is not a concern.

Although this work cannot bring conclusive proof regarding the thesis added by Palmerio (2017), that the intense vibratory regime was related to the accident with the VLS-1 in 1999, it does bring evidence that the PI-D used in the vehicle lacked the robustness to deal with vibrations and delays.



Further investigations of the LQG/LTR-based PMVC tuning, covering the Kalman filter design and analysis, and execution of the LTR procedure to achieve better gain and phase margins, will be the focus of future research related to this work, along with the development of a time-varying model of the VLS-1 to assess its performance and robustness during the atmospheric phase of flight.

CONFLICTS OF INTEREST

Nothing to declare.

AUTHOR'S CONTRIBUTION

Conceptualization: Silveira A; **Formal Analysis:** Silveira A; **Investigation:** Silveira A; **Resources:** Silveira A; **Validation:** Silveira A; **Data curation:** Silveira A; **Writing - Original Draft:** Silveira A; **Writing - Review & Editing:** Silveira A; Sagliano M; Seelbinder D; **Supervision:** Theil S; Fonseca Neto JV; Seruffo MCR; **Simulation:** Silveira A; Azonsivo R; Silva MM; Lima M; Silva D; Sodré LC; **Final approval:** Silveira A;

DATA AVAILABILITY STATEMENT

The data will be available upon request.

FUNDING

Conselho Nacional de Desenvolvimento Científico e Tecnológico 
Grant No: 311570/2025-9

ACKNOWLEDGEMENTS

The authors from the Universidade Federal do Pará acknowledge the 3rd Brazilian Aerospace Congress for supporting the publication costs.

DECLARATION OF USE OF ARTIFICIAL INTELLIGENCE TOOLS

The research, simulations, writing, and all content of this work were produced without the use of any artificial intelligence application.

REFERENCES

- Astrom KJ, Wittenmark B (2011) Computer-controlled systems: theory and design. 3rd ed. Mineola (NY), USA: Dover Publications.
- Doyle JC (1978) Guaranteed margins for LQG regulators. IEEE Trans Autom Control 23(4):756-757. <https://doi.org/10.1109/TAC.1978.1101812>

- Doyle JC, Stein G (1981) Multivariable feedback design: concepts for a classical/modern synthesis. *IEEE Trans Autom Control* 26(1):4-16. <https://doi.org/10.1109/TAC.1981.1102555>
- Doyle JC, Stein G (1979) Robustness with observers. *IEEE Trans Autom Control* 24(4):607-611. <https://doi.org/10.1109/TAC.1979.1102095>
- Nunes GFS, Carvalho APSD, Tavares Júnior AJA, Vito LGR, Sakamoto JMS, Oliveira NMF (2022) Simulation of an interferometric fiber optic gyroscope applied to a rocket model. Paper presented 2022 Simpósio de Aplicações Operacionais em Áreas de Defesa. ITA; São José dos Campos, Brazil. https://www.sige.ita.br/aiovg_videos/simulation-of-an-interferometric-fiber-optic-gyroscope-applied-to-a-rocket-model/
- Palmerio AF (2017) Introdução à tecnologia de foguetes. 2nd ed. São José dos Campos (SP): SindCT. [accessed Aug 23 2026]. <https://sindct.org.br/sindct/comunicacao/livros-e-cartilhas/introducao-a-tecnologia-de-foguetes/>
- Ramos FO (2011) Automation of H_∞ controller design and its observer-based realization (doctoral dissertation). São José dos Campos/Toulouse: Instituto Nacional de Pesquisas Espaciais/Institut Supérieur de l'Aéronautique et de l'Espace. https://oasisbr.ibict.br/vufind/Record/INPE_4f67369aef6932e3293f29199f60abcd
- Retro-Space-HD (2022) VLS-1 V2 Launch Failure - Solid Fuel Brazilian Rocket, 1999, Alcantara Space Center. [accessed Aug 23 2026]. <https://www.youtube.com/watch?v=6kx7EHqtG6Q>
- Seborg DE, Edgar TF, Mellichamp DA (2003) Process dynamics and control. 2nd ed. Hoboken (NJ), USA: John Wiley & Sons, Inc.
- Silva AG (2014) Análise e projeto de sistemas de controle de atitude para o veículo lançador de satélites (VLS) (master's thesis). São José dos Campos: Instituto Nacional de Pesquisas Espaciais. In Portuguese. <http://mtc-m21b.sid.inpe.br/attachment.cgi/sid.inpe.br/mtc-m21b/2014/03.06.16.40/doc/publicacao.pdf>
- Silva FO, Borges RA, Trofino A, Oliveira VA, Ferreira AMC (2019) Tuning techniques evaluation for satellite launch vehicle attitude controllers. *J Aerosp Technol Manag* 11:e2419. <https://doi.org/10.5028/jatm.v11.1004>
- Silveira A, Santos M, Brandão AS, Ribeiro M, Paiva EC (2020) Design and real-time implementation of a wireless autopilot using multivariable predictive generalized minimum variance control in the state-space. *Aerosp Sci Technol* 105:106053. <https://doi.org/10.1016/j.ast.2020.106053>
- Silveira A (2025) Attitude control and VLS-1 rocket public benchmark model [Internet]. MATLAB Central File Exchange. [accessed Aug 23 2026]. <https://www.mathworks.com/matlabcentral/fileexchange/176313-attitude-control-and-vls-1-rocket-public-benchmark-model>
- Stevens BL, Lewis FL, Johnson EN (2016) Aircraft control and simulation: dynamics, controls design, and autonomous systems. 3rd ed. Hoboken (NJ), USA: John Wiley & Sons, Inc. <https://doi.org/10.1002/9781119174882>
- Wie B, Du W, Whorton M (2012) Analysis and design of launch vehicle flight control systems. Paper presented 2012 AIAA Guidance, Navigation and Control Conference and Exhibit. AIAA; Minneapolis, USA. [accessed Aug 23 2026]. <https://arc.aiaa.org/doi/abs/10.2514/6.2008-6291>
- Yamada AFCS, Kienitz KH, Ramos FO (2024) Robust attitude control of a flexible launch vehicle subjected to wind disturbances. [accessed Aug 23 2026]. https://www.sba.org.br/cba2024/papers/paper_1406.pdf