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Neural Networks Modelling for Aircraft Flight Guidance Dynamics

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Abstract: *The sustained increase of the air transportation sector over the last decades has led to traffic saturated situations, inducing higher costs for airlines and important negative impacts for airport surrounding communities. The efficient management of air traffic supposes that aircraft trajectories are fully mastered and their impacts can be accurately forecasted. Inversion of aircraft flight dynamics, which are essentially nonlinear, appears necessary. Aircraft flight dynamics is shown to be differentially flat, which is a property that has enabled the development of new numerical tools for the management of complex nonlinear dynamic systems. However, since in the case of aircraft flight dynamics this differential flatness property is implicit, a neural network is introduced to deal with its numerical inversion. Results related to the developed neural network training are displayed, while potential uses of the proposed tool are discussed.*

Keywords: *Neural networks, Differential flatness, Aircraft flight dynamics.*

INTRODUCTION

The sustained increase of air transportation over the last decades has led to traffic-saturated situations. To manage safely and efficiently such traffic, new maneuvering capabilities are needed on board civil aviation aircraft to perform advanced 4D trajectories, while predictive tools, given reference 4D trajectories, are necessary to estimate accurately their impacts in terms of burned fuel and noise emissions. Both objectives require the ability of performing aircraft flight dynamics inversion. Differential flatness, a concept introduced by the school of Fontainebleau (Fliess *et al*., 1995), has provided new opportunities to design advanced management and supervision schemes for nonlinear systems. According to this theory, given the desired trajectory for what is called a flat output, it becomes straightforward to derive the corresponding input. Some authors have already given some insight into the differential flatness of aircraft flight dynamics (Lavigne *et al*., 2003; Martin, 1992), while others have considered its potential applications to aircraft trajectory management (Lu *et al*., 2004; 2008).

In this communication, firstly, a simplified proof of the implicit differential flatness of flight guidance dynamics is displayed. Secondly, a feed-forward neural network structure is developed to invert the flight guidance dynamics. Issues related to the effective training of such structure are discussed, and numerical results related to a reference aircraft model are displayed. These results show that the proposed approach allows the identification of flatness property of flight guidance dynamics, resulting in a numerical tool for new aircraft trajectory management applications.

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DIFFERENTIAL FLATNESS OF NONLINEAR SYSTEMS

Two definitions of differential flatness are introduced: one related to systems for which causal relationships of interest are analytically displayed, and another one where these causal relationships are introduced through implicit functions. A general nonlinear system whose dynamics are given by Eq. 1:

$$\underline{\dot{X}} = F(\underline{X},\underline{U})\ \underline{X} \in \boldsymbol{R}^n, \underline{U} \in \boldsymbol{R}^m \tag{1}$$

where: F is a smooth mapping, explicitly flat with respect to the output vector $\underline{Z}$, if $\underline{Z}$ is an m^{th} order vector, which can be expressed analytically as a function of the current state, the current input and its derivatives and also such as the state and the input vectors can be analytically expressed as a function of $\underline{Z}$ and its derivatives. There are smooth mappings G_x, G_u and G_z as in Eq. 2:

$$\underline{Z} = G_z(\underline{X},\underline{U},\ldots,\underline{U}^{(p)}) \tag{2a}$$

$$\underline{X} = G_x\left(\underline{Z},\underline{\dot{Z}},\ldots,\underline{Z}^{(q)}\right) \tag{2b}$$

$$\underline{U} = G_U\left(\underline{Z},\underline{\dot{Z}},\ldots,\underline{Z}^{(q+1)}\right) \tag{2c}$$

where: p and q are integer numbers. Vector $\underline{Z}$ is called a flat output for the nonlinear system. Although there is no systematical way to determine the flat output, the components of the flat output usually possess some physical meaning.

The explicit flatness property is of particular interest for the solution of a management problem when a meaningful flat output can be related to its objectives, for instance, in many situations, the management problem can be formulated as a flat-output trajectory tracking problem. However, for many systems, no complete analytical models are available to describe their full dynamics. Some of their components make use of input-output numerical tables derived both from theory and from experimental data. In these cases, the available theory provides, in general, the main mathematical properties of these implicit functions, while experimental data are used to build accurate input output numerical devices. This happens, for instance, when flight dynamics modeling is considered either for control or simulation purposes, since in practice the involved aerodynamic coefficients are obtained through interpolation across large sets of look-up tables.

A nonlinear system given by a general implicit n^{th} order differential representation (Eq. 3):

$$F(\underline{X},\underline{\dot{X}},\underline{U}) = 0,\ \underline{X} \in R^n, \underline{U} \in R^m \tag{3}$$

where: F is a regular implicit mapping with respect to $\underline{\dot{X}}$, which is said implicitly flat over an interior non-empty domain $\Delta \subseteq R^{n+m}$ if it is possible to find an m^{th} order vector $\underline{Z}$ that meets condition (Eqs. 1 and 2a) and condition in Eq. 4 (Lévine, 2011):

$$G(\underline{X},\underline{U},\underline{Z},\underline{\dot{Z}},\ldots,\underline{Z}^{(r)}) = \underline{0} \tag{4}$$

where: G is locally invertible over Δ with respect to $\underline{X}$ and $\underline{U}$, r is an integer. Again, vector $\underline{Z}$ is said to be a flat output. The local invertibility of G is guaranteed if the determinant of the Jacobian of G is not zero, according to the theorem of implicit functions, like in Eq. 5 (Lévine, 2011):

$$\det(\partial G/\partial(\underline{X},\underline{U}) \neq 0 \tag{5}$$

In this case, given a trajectory of the flat output $\underline{Z}$, it is possible to map it numerically into the input space to derive corresponding control signals, so that one of the more interesting properties of differentially flat systems is still maintained.

FLIGHT GUIDANCE DYNAMICS AND DIFFERENTIAL FLATNESS

In this study, only the guidance dynamics of transportation aircraft, i.e., the temporal trajectory followed by its center of gravity, is considered. It is assumed that the aircraft is equipped with a basic augmentation system and autopilot, which deal efficiently with its fast dynamics and controls its attitude angles (θ,ϕ,ψ) with respect to a local Earth frame, as well as its thrust regime (N1). Here, the flight variables , and N1 are taken as the inputs for the guidance dynamics. Figure 1 displays the resulting structure for the whole flight guidance dynamics.

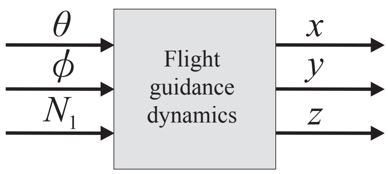

Figure 1. Input-output aircraft flight guidance structure.

Thus, in steady wind conditions, the flight guidance dynamics can be expressed in the aerodynamics reference frame as Eq. 6 (Lu *et al*., 2004):

$$\dot{x} = V_a \cos\psi \cos\gamma \tag{6a}$$

$$\dot{y} = V_a \sin\psi \cos\gamma \tag{6b}$$

$$\dot{z} = -V_a \sin\gamma \tag{6c}$$

where, γ is the ground path angle. The modulus of the inertial and the aerodynamic speeds are given by Eq. 7:

$$V_a = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} \tag{7a}$$

$$V_a = \sqrt{(\dot{x} - w_x)^2 + (\dot{y} - w_y)^2 + (\dot{z} - w_z)^2} \tag{7b}$$

where, w_x, w_y and w_z are the wind components.

Assuming that the sideslip angle β is very small and that θ and ϕ vary slowly, the following equations can be written as Eq. 8:

$$\dot{V}_a = (-D + T\cos\alpha)/m$$
$$-g(-\cos\alpha\sin\theta + \sin\alpha\cos\phi\cos\theta) \tag{8a}$$

$$\dot{\gamma}_a = (L + T\sin\alpha)/(mV_a)$$
$$-(g/V_a)(\sin\alpha\sin\theta + \cos\alpha\cos\phi\cos\theta) \tag{8b}$$

where: γ_a is the aerodynamic path angle which is related to γ by Eq. 9:

$$V_a\sin\gamma - w_z = V_a\sin\gamma_a \tag{9}$$

In coordinated-turn maneuver, the heading rate is related to the bank angle through the following relation (Eq. 10):

$$\dot{\psi} = (g/V_a)\tan\phi \tag{10}$$

The drag and the lift forces, D and L are, respectively, considered to be functions of altitude, z, airspeed V_a, and angle-of-attack, α. While the thrust T can be considered function of altitude z, airspeed and engine regime N1 (Eq. 11).

$$\begin{aligned} D &= D(z, V_a, \alpha) \\ L &= L(z, V_a, \alpha) \\ T &= T(z, V_a, N_1) \end{aligned} \tag{11}$$

For local guidance purposes, the flight-path angle γ is usually taken as the control parameter. When β is small, θ, ϕ, β and γ are related by Eq. 12:

$$\sin\gamma = \frac{(\sin\theta\cos\alpha - \cos\phi\cos\theta\sin\alpha)V_a - W_z}{V_a} \tag{12a}$$

which is reduced to:

$$\alpha = \theta - \gamma \tag{12b}$$

that is, in general, the case for a transportation aircraft. A unique solution in α corresponds to values of θ and ϕ.

Once $x(t)$, $y(t)$ and $z(t)$ are known, it is possible to use them and their derivatives to express all the guidance variables as follows.

By rearranging the kinematical equations (Eqs. 6a, 6b and 6c), it is possible to express Eqs. 13 and 14:

$$\gamma = -\sin^{-1}(\dot{z}/V_a) \tag{13}$$

$$\psi = \tan^{-1}(\dot{y}/\dot{x}) \tag{14}$$

The state variables V_a, γ and ψ can obviously be functions of inertial position of the aircraft, while the control variables satisfy the relations in Eq. 15:

$$\dot{V}_a - (-D + T\cos\alpha)/m$$
$$+g(-\cos\alpha\sin\theta + \sin\alpha\cos\phi\cos\theta) = 0 \tag{15a}$$

$$\dot{\gamma}_a - (L + T\sin\alpha)/(mV_a)$$
$$+(g/V_a)(\sin\alpha\sin\theta + \cos\alpha\cos\phi\cos\theta) = 0 \tag{15b}$$

$$\dot{\psi} - (g/V_a)\tan\phi = 0 \tag{15c}$$

where, α and γ_a can be expressed as functions of ϕ, θ and γ according to the relations in Eqs. 9 and 12.

The following notations are adopted for the position of the center of gravity of the aircraft and for the guidance inputs (Eq. 16):

$$\underline{Z} = (x, y, z)^T \text{ and } \underline{U} = (\theta, \phi, N_1) \tag{16}$$

Once the variables in Eq. 15 are replaced with their expressions in $\underline{Z}$, and their first two derivatives, these equations can be rewritten as in Eq. 17:

$$G_{N_1}(\underline{Z}, \underline{\dot{Z}}, \underline{\ddot{Z}}, \underline{U}, \underline{W}) = 0 \tag{17a}$$

$$G_\theta(\underline{Z}, \underline{\dot{Z}}, \underline{\ddot{Z}}, \underline{U}, \underline{W}) = 0 \tag{17b}$$

$$G_\phi(\underline{Z}, \underline{\dot{Z}}, \underline{\ddot{Z}}, \underline{U}, \underline{W}) = 0 \tag{17c}$$

Here, we suppose that either the wind speed vector is exactly known or null. Subsequently, the argument $\underline{W}$ will be deleted from the G_i functions. These implicit functions are locally invertible with respect to the input vector since, for normal flight conditions, the determinant of their Jacobian is not zero (Eq. 18):

$$\begin{vmatrix} \frac{\partial G_{N_1}}{\partial \theta} & \frac{\partial G_{N_1}}{\partial \phi} & \frac{\partial G_{N_1}}{\partial N_1} \\ \frac{\partial G_{\theta}}{\partial \theta} & \frac{\partial G_{\theta}}{\partial \phi} & \frac{\partial G_{\theta}}{\partial N_1} \\ \frac{\partial G_{\phi}}{\partial \theta} & \frac{\partial G_{\phi}}{\partial \phi} & \frac{\partial G_{\phi}}{\partial N_1} \end{vmatrix} \neq 0 \tag{18a}$$

Once this condition is equivalent to:

$$-\frac{\partial T}{\partial N_1}.(T + \left(\frac{\partial D}{\partial \alpha}.\sin\alpha + \frac{\partial L}{\partial \alpha}.\cos\alpha\right)).\frac{\partial \alpha}{\partial \theta} \neq 0, \tag{18b}$$

which is satisfied since, in general, all of its terms are definite positive.

Hence, $\underline{Z}=(x,y,t)^T$ is a flat output vector for the considered flight guidance dynamics. The time evolution of these flat outputs represents the trajectory followed by the center of gravity of the aircraft. Then, according to this theory, from the knowledge of this trajectory, it should be possible to find the corresponding inputs.

NEURAL NETWORK INVERSION OF THE FLIGHT GUIDANCE DYNAMICS

As a consequence of the flatness property, given a smooth reference trajectory for the flat outputs such as:

$$\underline{Z}_c(\tau) = (x_c(\tau), y_c(\tau), z_c(\tau))^T, \tau \in [t_0, t], \tag{19}$$

the corresponding reference input values at the instant t, $U_c(t)=(\phi_c(t), \theta_c(t) N_{1c}(t))^T$, are the solutions:

$$G_{N_1}(\underline{Z}_c(t), \dot{\underline{Z}}_c(t), \ddot{\underline{Z}}_c(t), \underline{U}_c(t)) = 0 \tag{20a}$$

$$G_{\theta}(\underline{Z}_c(t), \dot{\underline{Z}}_c(t), \ddot{\underline{Z}}_c(t), \underline{U}_c(t)) = 0 \tag{20b}$$

$$G_{\phi}(\underline{Z}_c(t), \dot{\underline{Z}}_c(t), \ddot{\underline{Z}}_c(t), \underline{U}_c(t)) = 0 \tag{20c}$$

Where, $\underline{Z}_c(t)$, $\dot{\underline{Z}}_c(t)$ and $\ddot{\underline{Z}}_c(t)$ are the current parameters.

In general, it will be very difficult to obtain an online numerical solution to this set of implicit equations, so it is useful to get an adequate numerical device to solve it. This adequacy can be specified mainly in terms of complexity and accuracy.

The differential flatness property of a dynamical system points out, in a reverse way, the causal relation between its inputs and eventually flat outputs. Since neural networks are particularly well adapted to reproduce causal relations, even in the case of very complex systems, it is interesting to try building a neural network with this objective. Once correctly trained, the neural network should be an input-output device where the inputs are provided by the reference trajectory, while the outputs are the nominal flight control parameters (Fig. 2).

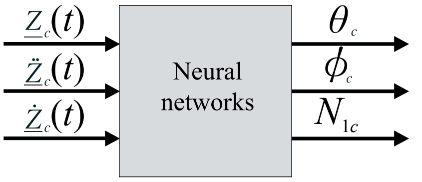

Figure 2. Reference input generator by neural networks.

Multilayer neural networks (MLNN) have been shown to be able to perform general function approximation through the selection of a neural network structure associated with a learning process (Haykin, 1994). The selection of the structure and of a learning algorithm strongly depends on empirical rules, while numerous candidate neural networks structures and learning algorithms are available. In order to achieve an acceptable accuracy and a sufficient generalization capability, a large amount of training data is necessary. Herein, the training data are composed of sets of trajectories for $\underline{Z}$ and $\underline{U}$, which can be provided from either flight test data or even from commercial flight data in which maneuvers are performed manually or by the autopilot engaged in a basic attitude-holding mode, in order that no guidance loop is active at that time (Mora-Camino, 1993).

Since, for modern aircraft, onboard navigation systems are able to estimate with good accuracy the current aircraft position, inertial speed and wind speed, their records can be used as a basis for the training of the neural network. A simulation model of a light aircraft, the Navion (Schmidt, 1998), with a piston-propeller engine and a basic controller for attitude holding, has been used for the generation of training data and validation purposes. Preliminary simulation results have been obtained in the case of maneuvers in the vertical plane. In this study, the conventional Error-Back-Propagation neural network with only one hidden layer has been selected to perform the inversion of flight guidance dynamics, although many other neural network structures have been investigated (Lu, 2005). Figure 3 displays some of the trajectories that have been considered to generate training data.

The structure of the retained neural network comprises seven inputs nodes, about 30 neurons in the hidden layer with a hyperbolic tangent activation function, and three output nodes with linear transfer functions. The seven

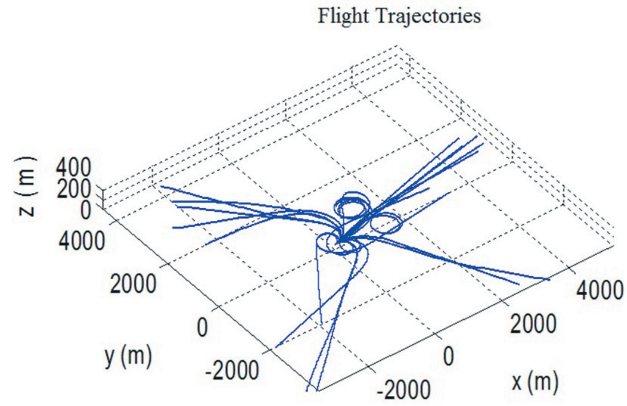

Figure 3. A set of training trajectories.

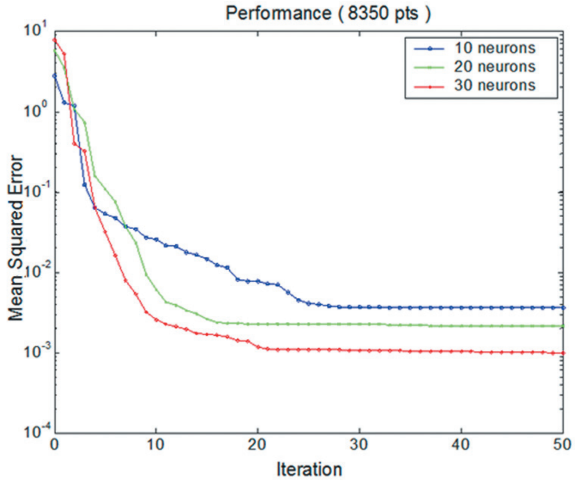

Figure 4. Training performance with different number of neurons in the hidden layer.

inputs are: altitude, three components of inertial velocity, and three components of inertial acceleration. The three outputs are the pitch θ, the roll ϕ and N_1, the engine regime. Figures 4 and 5 display examples of training performances for different structures and sizes of the training database.

Once the weights of a neural network have been optimized, the training of the neural network must be validated using an independent validation database. Table 1 displays an example of validation data performances, where $L(E)$ is the total mean square error of the neural network for a given inter-neurons weighting pattern and computed either over training data or validation data, S is the number of neurons in the hidden layer, and n is the number of effective connections between neurons.

Table 1. Example of training and validation data.

S	n	$L(E)$ training	$L(E)$ Validation
15	271	4.15×10^{-4}	3.87×10^{-3}
17	322	3.99×10^{-4}	2.72×10^{-3}
18	349	3.42×10^{-4}	1.20×10^{-3}
19	377	2.59×10^{-4}	1.75×10^{-3}
25	566	1.85×10^{-4}	3.52×10^{-3}

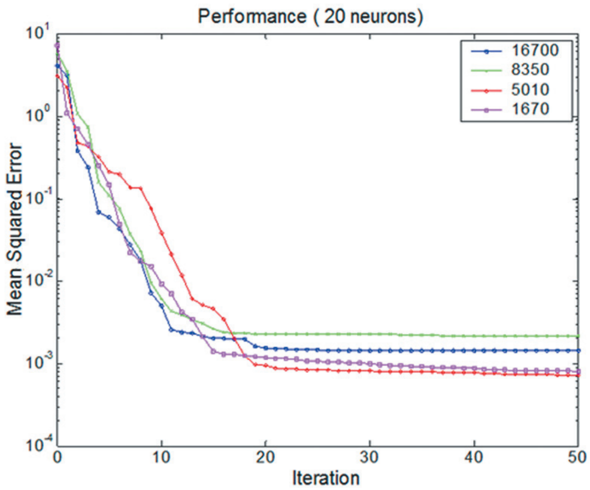

Figure 5. Training performance for different sizes of the training database.

A relevant validation of the neural network is obtained when, in nominal conditions (nominal flight model, no wind variation), the outputs of the neural networks are submitted as reference values to an autopilot operating in basic modes (attitude angles and engine regime tracking). Figure 6 displays the resulting reference and response trajectories of the simulated aircraft, as well as the trajectory error.

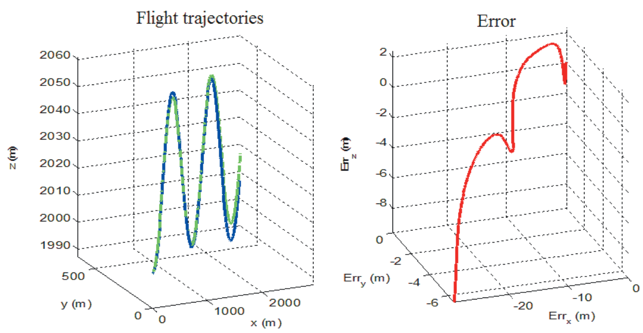

Figure 6. Example of open-loop control performance for flight trajectories.

CONCLUSIONS

This paper has shown how neural networks can be used to take profit of the implicit differential flatness property of the aircraft flight guidance dynamics. Differential flatness is a characteristic shared by many nonlinear systems and, in the case of complex systems, this property may appear in an implicit way. In this study, to make this property valuable for aircraft trajectory management, a feed-forward neural network structure has been proposed to invert the flight guidance dynamics. The performed numerical experiments show that by adopting classical neural networks structures and learning schemes, it is possible to achieve quite easily this objective. This approach allows an adequate identification of the inverse input-output relations associated with the flatness property of flight guidance dynamics.

This approach results in a useful numerical tool for many applications, such as: new trajectory tracking flight control structures as displayed in Fig. 7; and new sound exposure level computation schemes, such as the ones displayed in Fig. 8.

However, many issues remain open for further research works:

- definition of a minimum set of trajectories for generation of adequate training data;
- search for more efficient dynamics inversion neural network structures;
- setting of a clear balance between the neural inversion accuracy and the amount of computation for training; and
- generation of efficient reference trajectories.

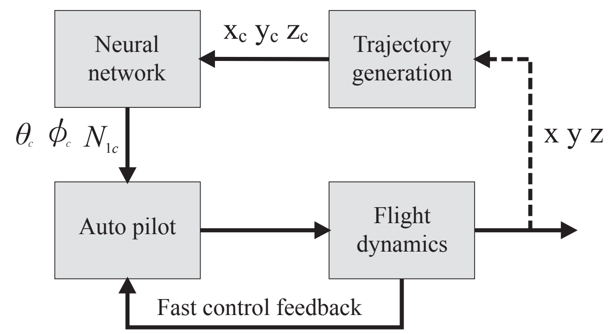

Figure 7. Trajectory tracking, including neural inversion.

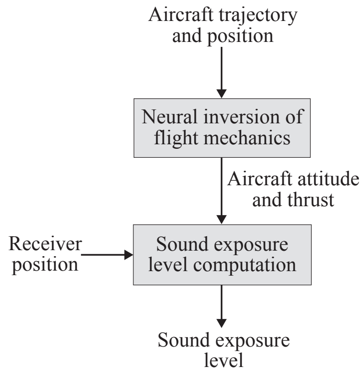

Figure 8. Noise exposure level estimation scheme.

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