EDITORIAL
Mathematics of Nanosatellites

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The role of small inexpensive satellites continuously grows in the modern space exploration. Their use can significantly reduce the cost of the mission. However, the control of such satellites is a challenge. One of the major issues here is that such satellites usually do not possess a complex attitude control system, and three-axis stabilization might be unavailable. As a consequence, the thrust vector of the orbit control system cannot be arbitrary oriented in space and rather involved mathematical methods are needed in order to compensate the control system's simplicity. The most frequently used simple and lightweight passive systems of one-axis stabilization are spin, passive magnetic or aerodynamic stabilization. In this case, one or two orbit control thrusters can be installed along the stabilized axis, so the orientation of the thrust vector at any given moment of time is determined by the orientation of stabilized axis. A very similar situation we face if the satellite uses the radiation pressure from the Sun as a motive force. Recently, a serious progress was achieved in the analysis and solution of the respective control problems. For example, dampers which use magnetic hysteresis rods in order to dissipate the energy of undesired angular motions — occurred during deployment or caused by perturbations — are used in attitude control systems of small satellites since the 1960s (Fischell and Mobley, 1964). The mathematical modeling of such systems is quite a difficult task, since the majority of existent hysteresis models result in differential equations with discontinuous right-hand side. The analysis of dynamics for attitude control systems with magnetic hysteresis dampers and optimization of their parameters have been done in Sarychev et al. (1988) and Guerman et al. (1989), and the results of these studies have been implemented in real missions (Ovchinnikov et al., 2000; Santoni and Zelli, 2009). However, these studies lacked an accurate theoretical basis for application of averaging methods to such problems. Recently, an adequate mathematical approach has been developed in Gama et al. (2011, 2013). It allows one to apply averaging technique to discontinuous systems and, therefore, rigorously justifies the main results from Sarychev et al. (1988) and Guerman et al. (1989). Another example concerns formation flying problems. A formation of nano- or picosatellites can perform a mission usually destined to big satellites. The main problem of a formation flying mission design is the maintenance of the required spatial configuration of satellites. Many works are focused on compensation of the satellites' relative drift caused by the J2 harmonic of the Earth's gravitational potential. In Guerman et al. (2012), a possibility to obtain a periodic relative motion of the chief and deputy satellites has been demonstrated for several types of single-input control, including the control oriented, along the geomagnetic field, and the control along an axis fixed in the absolute space. In each case, sufficient controllability conditions have been deduced. These conditions can be concisely formulated as follows: the vector of control direction should have non-zero components both in the orbital plane and along the normal to the orbit. To describe the relative motion of formation flying spacecraft, the Schweighart and Sedwick (2002) modification of the Hill-Clohessy-Wiltshire equations was used. In Guerman et al. (2014), the ideas from Guerman et al. (2012) were completed with a Newton-type method, which allows one to construct an exact trajectory and corresponding control law based on those obtained from the linearized model. Being implemented, such an iterative procedure significantly reduced the modeling error and confirmed the validity of results theoretically previously proved for the linearized system. Problems of orbital control become more involved if a gravitational field created by two bodies is considered, for example, orbital maneuver in the vicinity of Lagrangian

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points is a challenge. The MEC/MCTI/CAPES/CNPq/FAPs project "Orbital maneuvers of nano-satellites with constraint orientation of the trust", under the leadership of Antonio Fernando Bertachini de Almeida Prado and Georgi Smirnov, deals with the problems of the described type and it is focused on the development of analytical and computational methods of control system design. The most important expected results concern orbital corrections and de-orbiting of nano-satellites. The main mathematical tools used by the team members are averaging techniques, the Lyapunov function method combined with local controllability conditions, and the ideas from the theory of differential games. These instruments allow the team members to analytically solve some of the problems and to develop effective numerical methods when the problem cannot be analytically solved. For example,

the team members analytically and numerically studied the de-orbiting problem for a satellite equipped with an inflatable balloon for a large range of orbital inclinations. They developed a new de-orbiting control algorithm and obtained an estimate for the de-orbiting time. Those topics are helping to define new lines of research, as well as new topics for Thesis and Dissertations in the Space Engineering and Technology graduate school at the *Instituto Nacional de Pesquisas Espaciais* (INPE). We expect an increase in the production of papers in well known journals in the field in the next years, thanks to the state of the art research performed by the group of researches and students involved in this project. It is also expected an increase in the number of students looking to perform research in actual and important topics under development in those areas at INPE.

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