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Modeling elastic and thermal properties of 2.5D carbon fiber and carbon/SiC hybrid matrix composites by homogenization method

Abstract: *Advanced carbon fiber hybrid carbon-ceramic matrix composites are realizing their potential in many thermostructural components for aerospace vehicles. This work presents ab-initio predictions of elastic constants and thermal properties for 2.5D carbon fiber reinforced carbon-silicon carbide hybrid matrix composites, by using the homogenization technique. The homogenization technique takes properties of individual components of the composites (fiber and matrix) and characteristics of the geometrical architecture of the preform to perform calculations. Ab-initio modeling of mechanical and thermal properties is very attractive, especially during the material development stage, when larger samples may be prohibitively expensive or impossible to fabricate. Modeling is also useful when bigger samples would be prohibitively expensive or impractical. Thermostructural composites made of 2.5D preforms are easy to manufacture in relation to 3D preforms. Besides, 2.5D preforms are also resistant to thermo cycling and have high resistance to crack propagation in relation to ply stacked composites such as unidirectional (1D) and bidirectional (2D) structures. The calculations were performed by setting an overall carbon fiber volume fraction at 40, 45 and 50 for a 2D stacked composite, and volume fraction in Z-direction of 2, 4 and 6.*

Keywords*:* *Mechanical properties, Carbon-SiC composites, Elastic properties, Thermal properties.*

INTRODUCTION

Advanced fiber-reinforced composite materials have been widely used in various load bearing structures, from sporting goods to aerospace vehicles. The ever-increasing popularity of fiber-reinforced composites is largely due to their lightweight, high strength, and superior structural durability. The microstructure of composites plays a dominant role in forming all the composite properties, including failure mechanisms. In principle, property characterization of fibrous composites should be based on their precise microstructures. In practice, however, the true microstructures of the composites are often simplified in the characterization models, both geometrically and from the point of view of materials. The degree of simplification depends on the desired engineering accuracy. The theory of homogenization (Yan, 2003) is almost universally applied to characterize fibrous composite properties. Composite homogenization is a mechanics-based modeling scheme that transforms a body of a heterogeneous material into a constitutively equivalent body of a homogeneous continuum. A set of effective properties is obtained for the equivalent homogeneous continuum. Homogenization is an essential first step towards the design and analysis of larger scale and load-bearing structures in fibrous composites. The analysis of a multidirectional composite made of a single unidirectional fiber-reinforced lamina is a classical example. In this case, the unidirectional single lamina is first homogenized, each one with a set of effective properties. The laminate is then treated as a layered plate structure, capable of carrying globally applied thermomechanical loads.

Composites can be divided according to their temperature use. At high temperatures (T>500°C), only composites made with carbon or ceramic matrices and carbon fiber or any other ceramic fibers, as reinforcement, can be used in structural applications. Their outstanding thermomechanical properties overcome the shortcomings of ceramic or metal components. These materials have been largely developed on an empirical basis. Examples of thermostructural composites can be seen in Fig. 1. Carbon fiber reinforced carbon/silicon carbide hybrid matrix composites (CRFC-SiC) are considered to be one of the most potential thermostructural materials for aerospace components (e.g. thermal protection systems of reentry vehicles or rocket engine components) (Naislan, 2005; Bouquet, *et al.*, 2003; Christin, 2002). For example, carbon materials are suitable for high temperature structural

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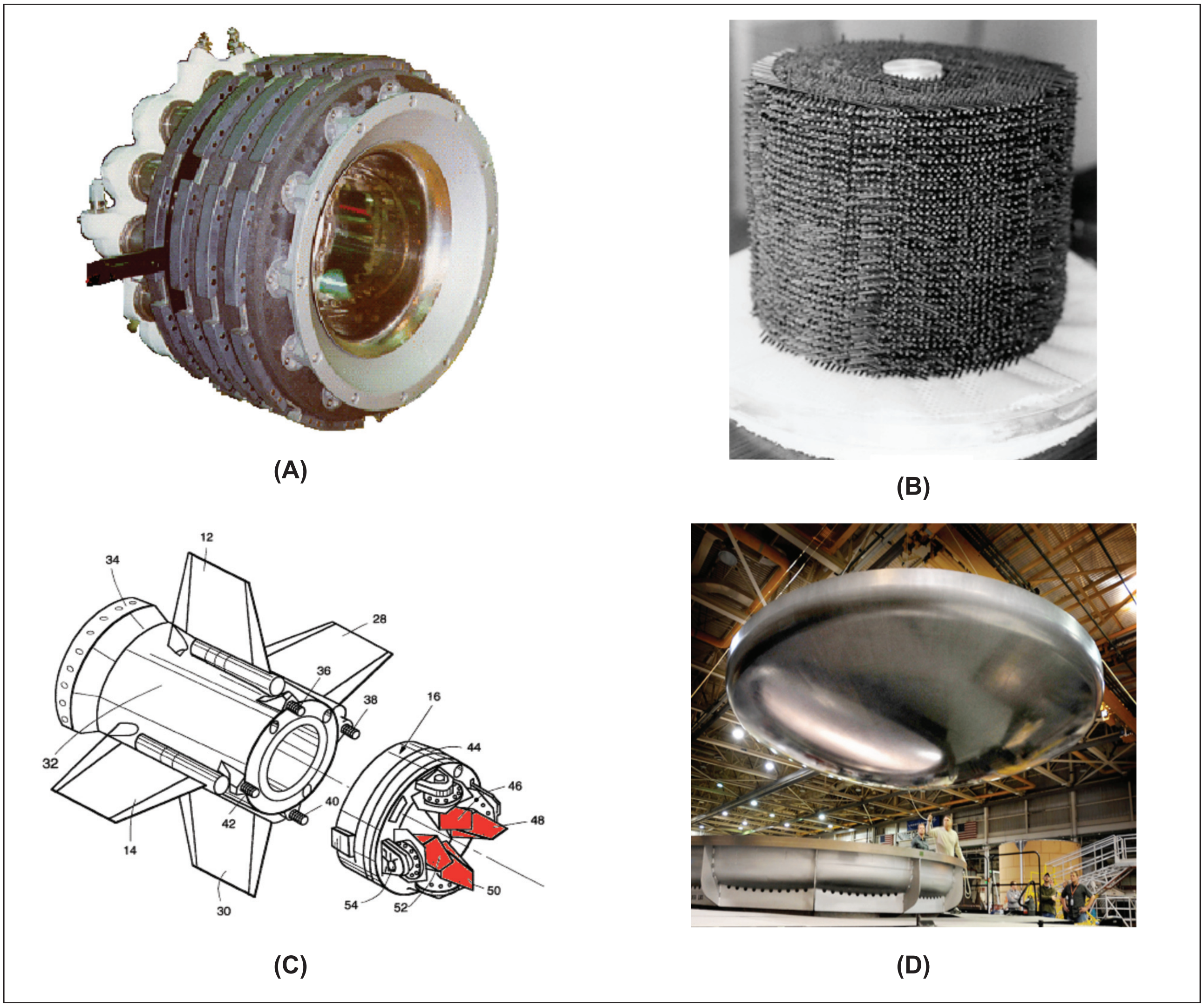


Figure 1: Applications of thermostructural composites. (A) Aircraft brake, (B) preform for a rocket nozzle, (C) vectorable vanes, (D) thermal protection system from Orion capsule.

materials because of their stable mechanical properties such as hardness and wear resistance in inert atmosphere. They are also light and not corrosive, but they exhibit a brittle like fracture. The traditional 2D preforms have high performance in-plane but they are susceptible to delamination. The multidirectional (3D, 4D, 5D, etc.) preforms exhibit an improved isotropy, good delamination resistance and thick part manufacturing capability (Hinders and Dickinson, 1997). By matching the good in plane properties of 2D composites and the high delamination resistance of multidirectional composites, it is possible to obtain a 2.5D reinforced composite. The 2.5D composites can be needled punched, Z-pinned or stitched, as depicted in Fig. 2. Commercially, thermostructural CRFC-SiC composites can be obtained by the routes as shown schematically in Fig. 3. In this case, the starting point is usually a porous carbon fiber reinforced carbon composite. The SiC matrix can be incorporated by the gas phase route (chemical vapor infiltration) from an organosilicon gas precursor, by infiltration of silicon in a porous carbon fiber reinforced carbon composite preform or by polymer impregnation and pyrolysis (PIP), using silicon-based polymers as precursors (Heindenreich, 2007; Guiomar, 1996; Interrante *et al.*, 2002). Considering investment, the PIP route is the simplest technique. The modeling of properties in this work considers the use of PIP method. Typically, CFRC-SiC composites have been made of bidirectional woven fabrics which are stacked together or wound to the desired thickness. Composites having 2.5D reinforcement differ from conventional 3D preforms by their fiber volume fraction. For 2.5D fiber reinforced composites fiber volume fraction in the Z-direction can be up to 10%. For a balanced orthogonal 3D composite, the fiber volume fraction can be up to 25% in each fiber axis direction. The insertion of Z direction reinforcement in bidirectional composites makes these composites in-plane crack resistant and allows them to endure many heat treatment cycles.

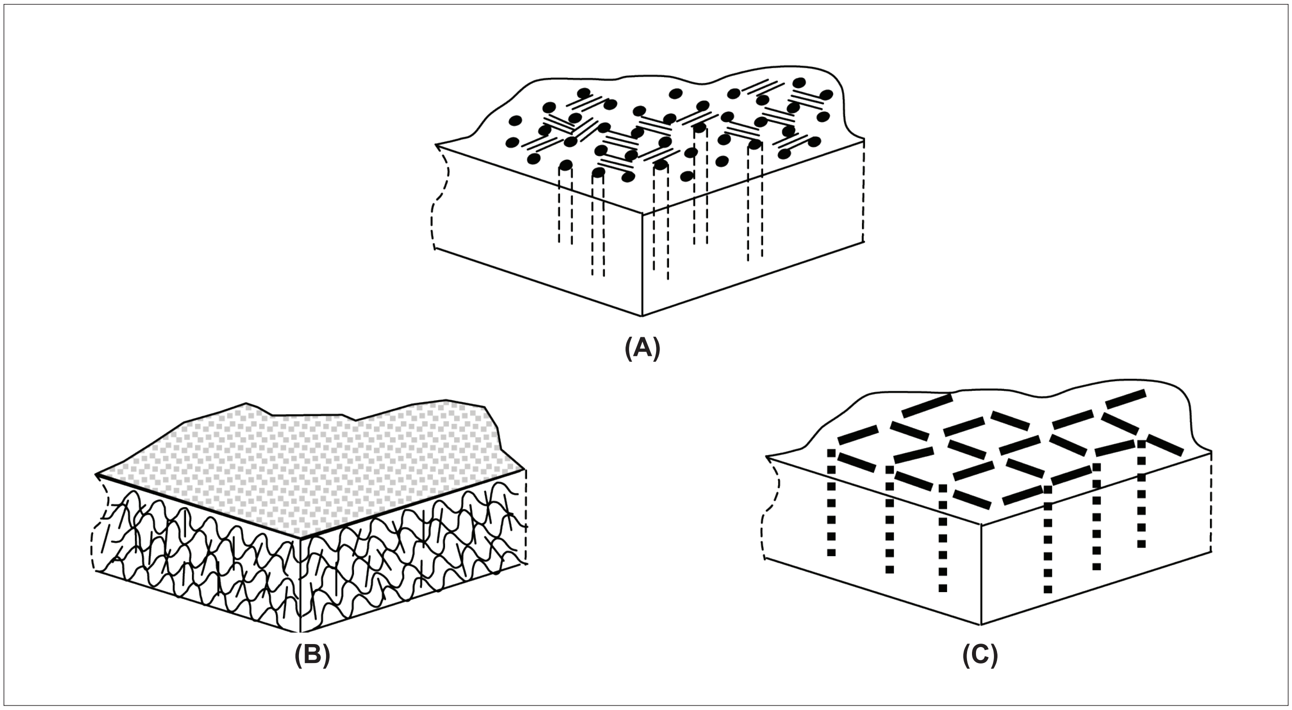


Figure 2: (A) Z-pinned preform, (B) needled punched preform, (C) stitched preform.

THE AVERAGE STIFFNESS COMPUTATIONAL METHOD

The properties of composites are designed by the selection of the fibers, their volume fraction, orientation, and architecture in the part. The problem is that the properties of the fibers are frequently altered by processing, and the properties of the matrix are even more sensitive to composite architecture and processing. The prediction and modeling of elastic constants for anisotropic materials and particularly for composites use analytical methods such as the Classical Lamination Theory or Finite Element Analysis (Hyer, 1998). Another way to predict elastic properties of composites take into account their microscopic nature, i.e. their intrinsic microstructure. The appropriate combination of the intrinsic properties of reinforcing fibers and matrix is then assumed. This approach is known as micromechanics, which is a study of the mechanical or thermal properties of composites, in terms of those of constituent materials. General assumptions in micromechanics of composites are: composite are macroscopically homogeneous and orthotropic, linearly elastic, initially stress free, and free of voids. Besides, it is assumed that there is complete bonding at the interface of the constituents and there is no transitional region between them. The displacements are continuous across the fiber matrix interphase (there is no interfacial slip). Fibers are homogeneous, linearly elastic, isotropic or orthotropic/transversely isotropic, regularly spaced, perfectly aligned, circular in cross-section and infinitely long in the longitudinal direction. Matrix is homogeneous, linearly elastic and isotropic. If any temperature effects are considered, the constituent material properties have to be known at a given temperature.

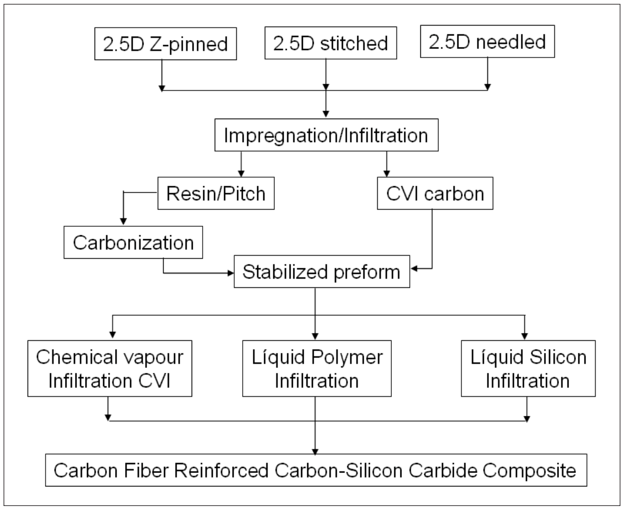


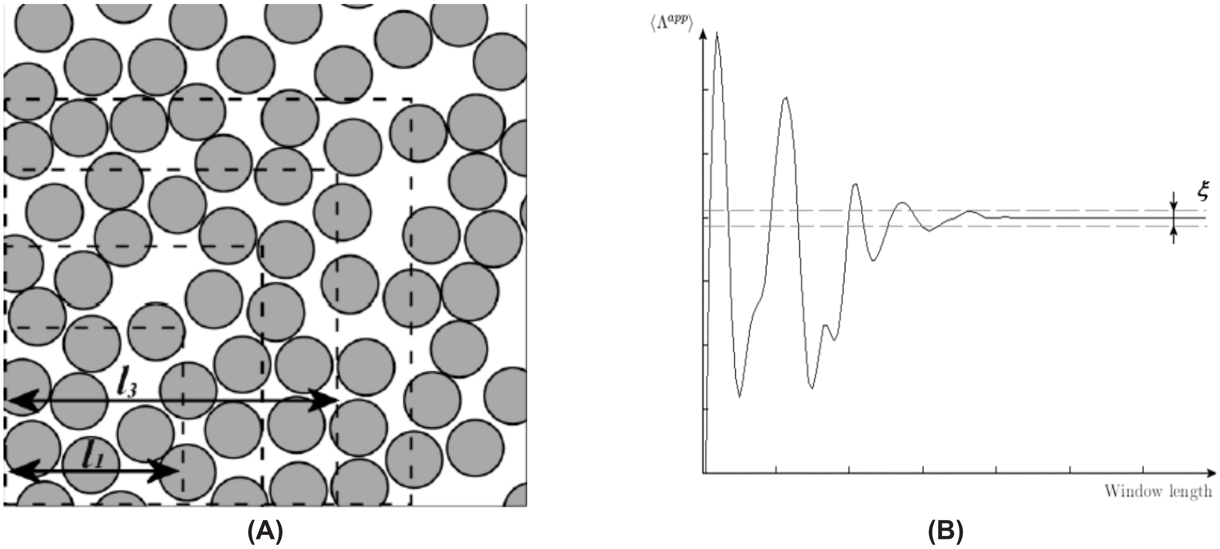
Figure 3: Commercial process routes used to obtain Carbon fiber reinforced carbon-silicon carbide composites.

The process of smoothing out quantities which vary on a microstructural lengthscale (lying between the atomic and macroscopic scales) is used in order to obtain some effective macroscopic properties. A set of effective properties is obtained for the equivalent homogeneous continuum. Of course, all matter is inhomogeneous at some scale, but frequently it is convenient to treat it as homogeneous. A good example is the continuum concept used in continuum mechanics. The Fig. 4A shows a representation of a unidirectional composite cross-section. An increase in the representative volume element (RVE), as showned by dotted lines, leads to a higher degree of homogeneity. Fig. 4B, on the other hand, shows schematically the effect of smoothing a generic asymptotic function that converges to a medium average value as the window length of the RVE is increased. This process of replacing an equation with a highly oscillatory coefficient for a homogeneous (uniform) coefficient is known as *homogenization*, which is linked to the subject of micromechanics. As a result, homogenization can be viewed as an extension of the continuum concept to materials which possess microstructure. The RVE is the analogue of the differential element in the continuum concept, which contains enough atom, or molecular structure to be representative of that material in homogenization and micromechanics. This RVE contains enough statistical information about the inhomogeneous medium in order to be representative of the material. Therefore, averaging over this element gives an effective property to fibrous composites, and so the homogenization is an essential first step towards the design and analysis of larger scale and load-bearing structures. The analysis of multidirectional composites made of unidirectional single lamina is a classical example. In this case, the unidirectional single lamina is first homogenized, each one with a set of effective properties. The laminate is then treated as a layered plate structure capable of carrying globally applied thermomechanical loads. For the reader, it is useful to review some points related to micromechanics for a better understanding of the average stiffness method, which is the basis of the elastic and thermal properties calculations.

The micromechanics approach is therefore used for elastic and thermal properties calculations. In the Classical Lamination Theory, for instance, the development of a procedure to evaluate stress and strain relations of composite laminates is fundamentally dependent on the fact that their thickness is much smaller than its plane dimensions. Typical thicknesses for individual composite layers can range from 0.10 to 0.25 mm. Consequently, composites having from 5 to 50 layers are considered thin plates and can be analyzed bearing in mind the simplifications of the thin plate theory. In the case of perfectly aligned fibers in a composite, as shown in Fig. 5, assuming linear elastic behavior and perfect adhesion, the Rule of Mixtures is applied, Eq. (1) (Matthews and Rawling, 1994). The equation is a representation of the composite longitudinal elastic modulus (E_{11}). The subscripts refer to the main reinforcement direction and the direction of applied stress, respectively. The transverse modulus (E_{22}) of the composite is defined by Eq. 2.

$$E_{11} = E_{fiber} \cdot V_{fiber} + E_{matrix} V_{matrix} \tag{1}$$

$$E_{22} = \frac{E_{fiber} \cdot E_{matrix}}{(V_{fiber} \cdot E_{matrix}) + (V_{matrix} \cdot E_{fiber})} \tag{2}$$



(A) (B)

Source: Thomas *et al*., 2008.

Figure 4: (A) Representative model of a unidirectional composite showing an increase in size of the representative volume element (RVE) and (B) asymptotic function model showing conversion to a medium value.

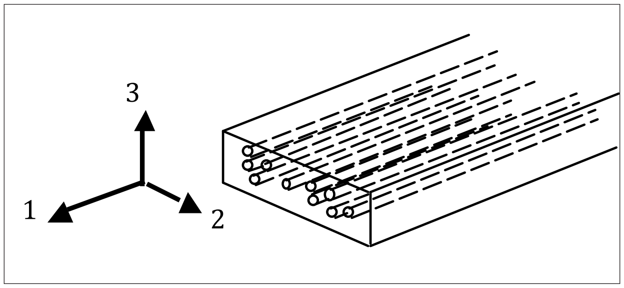


Figure 5: Schematic representation of a unidirectional (1D) composite and main axis.

Where E_{fiber} is the modulus of reinforcement, E_{matrix} is the matrix modulus, V_{fiber} is the volume fraction of fibers, V_{matrix} is the volume fraction of matrix.

The in-plane shear modulus (G_{12}) of a unidirectional composite is given by Eq. (3). Both the transverse and shear modulus are strongly influenced by the matrix modulus (G_{matrix}).

$$G_{12} = \frac{G_{fiber} \cdot G_{matrix}}{(V_{fiber} \cdot G_{matrix}) + (V_{matrix} \cdot G_{fiber})} \tag{3}$$

Where G_{fiber} is the shear modulus of the fiber, G_{matrix} is the matrix shear modulus, V_{fiber} is the fiber volume fraction, V_{matrix} is the matrix volume fraction.

For transversely isotropic materials, properties of the material in directions 2 and 3, from Fig. 5, are almost the same and are transversely isotropic to the direction 1. So, the following identities are valid: $E_{22} = E_{33}$, $\nu_{12} = \nu_{13}$, $G_{12} = G_{13}$, and:

$$G_{23} = \frac{E_{22}}{2 \cdot (1 + \nu_{23})} \tag{4}$$

Where ν_{12}, ν_{13} and ν_{23} are Poisson ratios, where the first subscript refers to the direction of stress and the second subscript refers to the direction of contraction.

The numerical procedure for the analysis of composites assumes the following hypotheses: (i) regular distribution of fibers in the tow and, (ii) regular assembly of fibers in the unidirectional composite rod. These hypotheses allow figuring out the problem of property estimation in the scope of micromechanics theory of periodic microstructure heterogeneous materials, corresponding to two levels of homogenization (Yan, 2003: Pastore and Gowayed, 1994; Gramoll, Freed and Walker, 2001). The first level refers to the fibers, which are analyzed independently by the rule of mixture model. Each unidirectional fiber forms the composite (Fig. 6A) having the fiber volume fraction similar to the packing density. So, the mechanical properties of the homogenized fiber (Fig. 6B) are obtained. The second level refers to "fictitious" multidirectional composite, having tows of fibers (rods) homogenized (Fig. 6C). The regularly spaced fibers in the composites allow to determine the representative volume element (Fig. 6D), and to evaluate the homogenized material (Fig. 6E) in terms of macroscopic quantities (global) (Pastore and Gowayed, 1994; Gramoll, Freed and Walker, 2001).

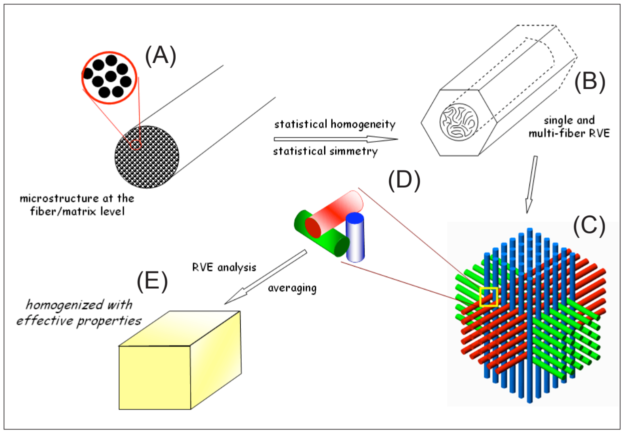


Figure 6: The hierarchical constitutive model: (A) unit cell, (B) composite rod, (C) representative unit of the fiber as unidirectional reinforcement, (D) preform representation and unit cell of the composite, (E) homogenized composite.

This model is described as "Fabric Geometry Model - FGM" (Pastore and Gowayed, 1994), which is based on the idea that the elastic properties of the composite can be calculated as a function of the relative proportion of the properties of the fibers, at specific directions, and the matrix. In summary, the composite is analytically divided in volume elements (unit cell) composed of composite rods. Each unidirectional reinforced rod has its own fiber volume fraction and from their vetorially oriented contributions forms the properties of the homogenized composite. The properties of each direction, represented by the composite rod in the unit cell, are calculated and the results for elastic constants refer to the local axis system (1-2-3), as shown in Fig. 7.

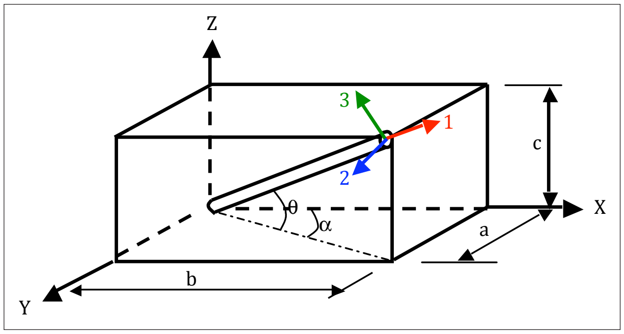


Figure 7: Local axis system (1, 2, 3) and global axis system (x, y, z) of coordinates, and 1, 2 and 3 are orthogonal.

The axis transformation from the stiffness matrix (C) and compliance (S) can be obtained by means of stress/strain transformation axis (Pastore and Gowayed, 1994; Gramoll, Freed and Walker, 2001). The resulting equation from axis transport for the stiffness matrix can be obtained as shown in Eq. (5).

$$[C_{global}]=[K][T][C_{local}][K] \tag{5}$$

Where $[C_{global}]$ is the matrix stiffness in the global axis system (x,y,z), [K] is the stress/strain transportion matrix, $[C_{local}]$ is the stiffness matrix in the local axis system (1-2-3), and T is the transformation matrix.

The transformation of constants from the local system to the global system is done by a transformation matrix composed of entities related to direction cosines (angles θ and α, respectively related to elevation and azimuth) from orientation of the rods. So, due to orthogonality only the Tε matrix is needed, which is represented in Eq. 6. The l_i, m_i and n_i are direction cosines components of the unit basis vectors associated with the principal axes of the fibrous reinforcement.

$$T_\varepsilon = \begin{vmatrix} l_1^2 & l_2^2 & l_3^2 & l_2 l_3 & l_1 l_3 & l_1 l_2 \\ m_1^2 & m_2^2 & m_3^2 & m_2 m_3 & m_1 m_3 & m_1 m_2 \\ n_1^2 & n_2^2 & n_3^2 & n_2 n_3 & n_1 n_3 & n_1 n_2 \\ 2m_1 n_1 & 2m_2 n_2 & 2m_3 n_3 & m_2 n_3 + m_3 n_2 & m_1 n_3 + m_3 n_1 & m_1 n_2 + m_2 n_1 \\ 2l_1 n_1 & 2l_2 n_2 & 2l_3 n_3 & l_2 n_3 + l_3 n_2 & l_1 n_3 + l_3 n_1 & l_1 n_2 + l_2 n_1 \\ 2l_1 m_1 & 2l_2 m_2 & 2l_3 m_3 & l_2 m_3 + l_3 m_2 & l_1 m_3 + l_3 m_1 & l_1 m_2 + l_2 m_1 \end{vmatrix} \tag{6}$$

In Fig. 7 the axis 1 is related to vector r_1, which is the unit vector associated with fiber axis (l_1, l_2, l_3), the axis 2 is related to vector r_2, which is the unit vector associated with #2 direction of the fiber (m_1, m_2, m_3), and axis 3 is related to vector r_3, which is the unit vector associated with #3 direction of the fiber (n_1, n_2, n_3). The vector $\mathbf{r_1}$ can be known by solving the geometrical relationship represented by Eq. (7).

The other two vectors can be known by solving through geometric relationships. Because they are mutually orthogonal, it can be proved that $\mathbf{r_1}.\mathbf{r_2} = 0$, $\mathbf{r_2}.\mathbf{r_3} = 0$ e $\mathbf{r_1}.\mathbf{r_3} = 0$. As they are direction cosines vectors, $||\mathbf{r_1}|| = ||\mathbf{r_2}|| = ||\mathbf{r_3}|| = 1$, and a, b and c are the edges of the unit cell.

$$r_1 = \frac{a}{\sqrt{a^2+b^2+c^2}}i + \frac{b}{\sqrt{a^2+b^2+c^2}}j + \frac{c}{\sqrt{a^2+b^2+c^2}}k \tag{7}$$

Where $(\text{diagonal})^2 = a^2 + b^2 + c^2$.

The contribution from each rod in the direction of the global axis system is done by the superposition of the stiffness matrix or flexibility matrix transposed for the global system, as shown by Eq. 8 (Pastore and Gowayed, 1994).

$$C_{comp} = \frac{1}{V_{fiber}} \sum_{i=0}^{n} V_{fiber-i} \cdot C_i \tag{8}$$

Where C_{comp} is the composite stiffness matrix, V_{fiber} is the total fiber volume fraction, $V_{fiber-i}$ is the relative fiber volume fraction from the i-esime unidirectional composite rod, C_i is the stiffness of the i-esime unidirectional composite rod and n is the number of unidirectional composite rods.

The thermal properties (thermal conductivity and coefficient of thermal expansion) were also calculated on the basis of average properties from the individual constituents of the composite by using the Fabric Geometry Model – FGM. During transient heat flow, transverse heat flow between the fiber and the matrix is expected to occur. This is the case for materials with significantly different thermal conductivities. If their interfaces are negligible in thickness, the longitudinal thermal conductivity of the composite will only be affected by the fibers and the matrix thermal conductivities, and will be unaffected by the fibers/matrix interface according to Eq. 9.

$$k_{c(//)} = k_{fiber}.V_{fiber} + k_{matrix}.V_{matrix} \tag{9}$$

Where $k_{c(//)}$ is the thermal conductivity parallel to the fiber direction, k_{fiber} and k_{matrix} are the thermal conductivity of the fibers and matrix, respectively and V_{fiber} and V_{matrix} are the volume fraction of the fibers and matrix, respectively. The thermal conductivity in the perpendicular direction of the composite is calculated by means of Eq. 10.

$$k_{c(\perp)} = k_{matrix(//)} \cdot \frac{\left[(1-V_{fiber}).\left(\dfrac{1-{k_{fiber(//)}}/{k_{matrix(//)}}}{1+{k_{fiber(//)}}/{k_{matrix(//)}}}\right)\right]}{\left[(1+V_{fiber}).\left(\dfrac{1-{k_{fiber(//)}}/{k_{matrix(//)}}}{1+{k_{f(//)}}/{k_{matrix(//)}}}\right)\right]} \tag{10}$$

Where $k_{c\,(\perp)}$ is the thermal conductivity perpendicular to the fiber direction, $k_{fiber\,(//)}$ is the thermal conductivity parallel to the fiber, $.k_{matrix(//)}$ is the thermal conductivity parallel to the matrix, and V_{fiber} and V_{matrix} are the volume fraction of the fibers and matrix, respectively.

For modeling the coefficient of thermal expansion, each phase is assumed homogeneous and isotropic and linearly elastic over a small range of volumetric strains. In the absence of the phase interaction, one may expect the coefficient of thermal expansion of a composite, with a reasonable approximation, to follow the simple rule of mixtures. However, because of the differences in the thermal expansivities of the phases (fiber and matrix), a state of micro-stress often exists between them, influencing the thermal expansion behavior of the body and giving rise to discrepancies. Thus, its thermal expansion coefficient does not follow the rule of mixtures (Thomas *et al.*, 2008; Mukerji, 1993).

Karadeniz and Kumlutas (2007) investigated the existing theories for prediction of coefficient of thermal expansion based on micromechanics and finite element analysis, considering the fiber volume fraction ranging from 10 to 90% for the studied composites (Mukerji, 1993). The theories agreed quite reasonably for calculation of the thermal expansion coefficient in the main fiber axis direction (α_1), at least for ordinary unidirectional polymer matrix composites, even compared with sophisticated modeling techniques, such as finite element calculations, for a range of fiber volume fractions (Mukerji, 1993). Nevertheless, for the transverse direction (α_2), the rule of mixtures seems to have a better agreement with finite element calculations than other micromechanical theories. The coefficient of thermal expansion for composite materials is a function of stress and strain in the matrix and fiber, according to Eq. (11).

$$\begin{aligned} \sigma_{fiber} &= E_{fiber}(\varepsilon_{fiber} - \alpha_{fiber}\Delta T) \\ \sigma_{matrix} &= E_{matrix}(\varepsilon_{matrix} - \alpha_{matrix}\Delta T) \end{aligned} \tag{11}$$

Considering $\varepsilon_{comp} = \varepsilon_{matrix} = \varepsilon_{fiber}$, in the fiber direction, the Eq. (12) is obtained.

$$\varepsilon_{fiber} = \frac{\alpha_{fiber}.V_{fiber}.E_{fiber} + \alpha_{matrix}.V_{matrix}E_{matrix}.\Delta T}{E_{fiber}.V_{fiber} + E_{matrix}V_{matrix}} \tag{12}$$

If there is free expansion in the direction of the fiber, $\varepsilon_1 = \alpha_1.\Delta T$, where α_1, it is according to Eq. (13) as follows:

$$\alpha_1 = \frac{\alpha_{fiber}.V_{fiber}.E_{fiber} + \alpha_{matrix}.V_{matrix}.\Delta T}{E_{matrix}.V_{fiber} + E_{matrix}.V_{matrix}} = \frac{1}{E_1}\alpha_{fiber}.V_{fiber}.E_{fiber} + \alpha_{matrix}.V_{matrix}.E_{matrix} \tag{13}$$

For the transverse direction to the fiber axis, the coefficient of thermal expansion (α_2) is obtained as shown in Eq. (14), as follows:

$$\alpha_2 = (1+\nu_{fiber})\alpha_{fiber}.V_{fiber} + (1+\nu_{matrix})\alpha_{matrix}.V_{matrix} - \alpha_1.\nu_{12} \tag{14}$$

RESULTS FOR AVERAGE STIFFNESS 2.5D CARBON FIBER REINFORCED C/SIC HYBRID MATRIX COMPOSITES

For the prediction of the mechanical properties of 2.5D CFRC-SiC composites through the micromechanics method, it is necessary initially to establish representative properties of the carbon fiber, the carbon matrix and the silicon carbide matrix. Mechanical properties of carbon fibers are well documented in the literature (Peebles, 1994; Inagaki, 2001; Asakuma *et al.*, 2003). Table 1 shows the properties of carbon fiber, SiC matrix and carbon matrix found in literature. The best value for the properties of a carbon matrix can rely on synthetic graphite mechanical and thermal properties (Inagaki, 2001). Synthetic graphites are obtained by a controlled pyrolysis process, up to 2500°C, from a mixture of pitch and coke. In relation

Table 1: Properties of carbon fiber, SiC matrix and carbon matrix reported in the literature and from manufacture’s data

Temperature (°C)	**Carbon fiber**			**SiC matrix***			**Carbon matrix**		
	RT	**1000**	**1200**	**RT**	**1000**	**1200**	**RT**	**1000**	**1200**
Elastic modulus (GPa)									
Longitudinal	230	230	230	225	225	225	10	10	10
Transversal	20	20	20	225	225	225	10	10	10
Shear modulus (GPa)	12	12	12	75	75	75	4.2	4.2	4.2
Poisson Ratio ν		0.18			0.14			0.18	
Density (g/cm^3)		1.78			2.55			1.80	
k (W/m.K)									
Longitudinal		8.40			1.768		120	60	55
Transversal		0.84			1.377				
α (10^{-6}/°C)									
Longitudinal		18			0.164		-0.6	1.5	2.0
Transversal		1.2							

*calculated based on Nicalon CG fiber.

Source: Peebles (1994), Inagaki (2001), Asakuma (2003).

to other carbon matrix materials obtained from gas phase deposition or thermoset resin pyrolysis, the mechanical properties of synthetic graphite are lower. A reasonable estimation of the elastic properties of a carbon matrix based on pitch coke, which is nearly the same of other graphitic materials, was reported by Asakuma (Asakuma *et al.*, 2003).

Silicon carbide can be obtained by a number of techniques, and the most common are hot pressing and chemical vapor deposition. In this manner, their properties can vary greatly depending on the method of preparation and the final microstructure obtained. But only few works have mentioned properties of silicon carbide obtained by any of the silicon polymer pyrolysis methods, possibly because of the difficulties of getting representative samples for traditional mechanical tests, such as tensile strength. On the other hand, mechanical properties for hot pressed silicon carbide only rely on flexural strength or compressive strength, and respective modulus, mainly because covalent ceramics are not suitable for use under tensile loads. Reported values for flexural strength of hot pressed SiC can be up 450 MPa (Mukerji, 1993). Somiya and Inomata (1992) found for a CVD-SiC material, tested under three-point bending values in the range of 200-690 MPa, and in this work a tensile strength of 590 MPa was also reported. The best combination of properties was obtained for CVD-SiC having a 1.5-μm grain size (Munro, 1997).

The Young's modulus of typical hot-pressed SiC materials is reported in the range of 275-465 GPa. The characterization of tensile strength of SiC, as any other covalent ceramic, is hardly difficult because they are prone to surface defects due to machining and by other inadequate specimen preparation procedures which leads to misleading measurements. Besides, tensile strength of SiC has a low value compared to their flexural and compressive strength. This is the reason why the uses and the mechanical characterization of SiC are practically done under flexure or compressive loading. In fact, the tensile tests lead to conservative results in relation to the other types of tests, such as flexure tests, and, as a consequence, the tensile strength can be much lower than flexural strength. Munro (1997), for instance, found a tensile strength of ~250 MPa for a hot pressed α-SiC, from 25°C to 1400°C.

The best known SiC material obtained from silicon polymer pyrolysis is the Nicalon fiber (Yajima *et al.*, 1979). Although the Nicalon SiC fiber varies in composition, it represents the properties of a typical polymer ceramic silicon carbide matrix. The chemical composition of Nicalon ceramic grade fiber has typically ~65% SiC, ~15% carbon and ~20% SiO_2. At room temperature, its Young's modulus is ~200 GPa and the tensile strength is 3.0 GPa. If considered isotropic, the shear modulus of Nicalon ceramic grade fiber is ~87 GPa, considering the relation $E=2G(1+\nu)$, but experimental results show values near 75 GPa for fibers processed at 1200°C. Increasing the test temperature up to 1200°C causes no noticeable changes on Young's modulus and in the shear modulus (Villeneuve and Naslain, 1993). Studies conduced by Soraràu, Dallapiccola and Dándrea (1996) on SiC_xO_y glasses synthesized by the sol-gel method found a Young's modulus of 115 MPa for these materials. For SiC_xO_y glasses, the properties beyond 1200°C fall rapidly. The prediction of elastic and thermal properties for carbon fiber reinforced carbon-SiC hybrid matrix was done by taking the properties of Table 1, for carbon fiber, SiC matrix and carbon matrix. In this case, the properties of the SiC matrix were considered as similar to the SiC fiber which is the best approximation for a typical amorphous polymer ceramic Si-C-0 matrix.

For composites made with ceramic matrices, processed by the polymer impregnation method, porosity in the range of 5 to 20% is usually attained (Rice, 1999). In this work, for comparison purposes and for good of prediction, it is considered a composite having ~5% volume porosity, and carbon fiber volume fraction of 40, 45 and 50%. The carbon matrix volume fraction will vary from 35, 30, and 25%, respectively to carbon fiber volume fraction, as shown in Table 2. The volume of SiC polymer ceramic matrix is kept, also for comparison purposes, at 20% volume fraction. In the calculations, the out-of-plane Z fiber addition is considered to be uniformly distributed throughout the composite. So, properties of the CFRC-SiC composite can be varied by taking the relative proportions of carbon and silicon carbide, according to the rule of mixture, as shown in Table 2. It is important to point out that mechanical properties are approximately constant from room temperature to 1200°C for these composites.

From input parameters from Table 1 and Table 2, the output parameters processed by the FGM model such as local resilient matrix and stiffness matrix, and global stiffness matrix, are obtained, as shown in Table 3, related to the direction cosines of a unidirectional composite. Results from Table 3 leads to elastic and thermal properties showed in Table 4. The elastic modulus for bidirectional Carbon Fiber Reinforced C/SiC Hybrid Matrix was also calculated, for comparison purposes.

It is hardly difficult to find properties of C-SiC composites mainly because differences in the processing routes, type of carbon fiber used, level of porosity in the composite and C/SiC matrix ratio. Besides, fiber volume fraction can vary from one material to another and mechanical properties are mostly found as manufacture's data sheet rather than a rigorous scientific research. Moreover, for the

Table 2: Properties of hybrid matrices of carbon/silicon carbide related to volume fractions of carbon and silicon carbide used for modeling the properties of CFRC/SiC-modified composites

Total carbon fiber volume fraction (%)	40	45	50
Carbon matrix volume fraction (%)	35	30	25
SiC matrix volume fraction (%)	20	20	20
Void volume fraction (%)	5	5	5
C/SiC matrix ratio	65/35	60/40	55/45
Density (g/cm^3)	2.06	2.10	2.14
Tensile strength (MPa)	145	165	180
Elastic Modulus (GPa)	82	96	105
Shear modulus* (GPa)	28	32	35
Poisson ratio	0.17	0.16	0.15
k (W/m.K)	79	72	66
α ($10^{-6}/^{o}C$)	1.00	1.04	1.17

*calculated by $E=2G(1+\nu)$.

Table 3: Direction cosines, transformation matrix, local resilient matrix, local stiffness matrix and composite stiffness matrix for a CFRC-SiC unidirectional composite model

Direction cosines →	**1**	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**1**

Transformation matrix

$$\left\{\begin{matrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{matrix}\right\}$$

Local resilient matrix

$$\left\{\begin{matrix} 0.00597015 & -0.00110448 & -0.00110448 & 0 & 0 & 0 \\ -0.00110448 & 0.0297619 & -0.00748348 & 0 & 0 & 0 \\ -0.00110448 & -0.00748348 & 0.0297619 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.047619 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0744908 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.047619 \end{matrix}\right\}$$

Local stiffness matrix

$$\left\{\begin{matrix} 170.6299 & 8.4592 & 8.4592 & 0 & 0 & 0 \\ 8.4592 & 36.2871 & 9.4381 & 0 & 0 & 0 \\ 8.4592 & 9.4381 & 36.2871 & 0 & 0 & 0 \\ 0 & 0 & 0 & 21.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 13.4245 & 0 \\ 0 & 0 & 0 & 0 & 0 & 21.0000 \end{matrix}\right\}$$

Composite stiffness matrix

$$\left\{\begin{matrix} 170.6299 & 8.4592 & 8.4592 & 0 & 0 & 0 \\ 8.4592 & 36.2871 & 9.4381 & 0 & 0 & 0 \\ 8.4592 & 9.4381 & 36.2871 & 0 & 0 & 0 \\ 0 & 0 & 0 & 21.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 13.4245 & 0 \\ 0 & 0 & 0 & 0 & 0 & 21.0000 \end{matrix}\right\}$$

Table 4: Properties of unidirectional (1D) carbon fiber reinforced C/SiC hybrid matrix

Total carbon fiber volume fraction (%)		**40**	**45**	**50**
C/SiC matrix ratio		**65/35**	**60/40**	**55/45**
E_{11}		141	156	167
E_{22}		36	35	33
E_{33}		36	35	33
G_{12}		20	21	21
G_{23}		14	14	13
G_{13}		20	21	21
υ_{12}		0.18	0.18	0.18
υ_{13}		0.18	0.18	0.18
υ_{23}		0.25	0.25	0.25
k (W/mK)	k_{11}	57	50	45
	k_{22}	35	30	25
	k_{33}	35	30	25
α (10^{-6}/°C)	α_{11}	0.12	0.70	1.05
	α_{22}	0.68	0.34	0.63
	α_{33}	0.68	0.34	0.63

Table 5: Properties of bidirectional (2D) carbon fiber reinforced C/SiC hybrid matrix

Total carbon fiber volume fraction (%)		**40**	**45**	**50**
C/SiC matrix ratio		**65/35**	**60/40**	**55/45**
E_{11}		89	96	98
E_{22}		89	96	98
E_{33}		37	36	37
G_{12}		20	21	21
G_{23}		17	17	18
G_{13}		17	17	18
υ_{12}		0.07	0.06	0.06
υ_{13}		0.23	0.22	0.21
υ_{23}		0.23	0.22	0.21
k (W/mK)	k_{11}	46	40	38
	k_{22}	46	40	38
	k_{33}	35	30	28
α(10^{-6}/°C)	α_{11}	0.56	0.92	0.97
	α_{22}	0.56	0.92	0.97
	α_{33}	0.04	0.54	0.60

majority of applications, the Z reinforcement has the main purpose of holding together the bidirectional stacks of fiber reinforcement fabrics, improving interlaminar shear and through-the-thickness fracture toughness than truly improve through-the-thickness elastic or thermal properties. So, with a reasonably approximation, the calculated properties using the Fabric Geometry Model can be compared with 2D CRFC-SiC composites. For instance, Wang *et al.* (2008) found for a 2D CFRC-SiC composite processed by CVI technique, having 40%/volume of fibers and 13% porosity, a Young's modulus of 95±9 GPa. Arendts and Maile (1998) reported a Young's modulus of 70-80 GPa for a 2D CRFC-SiC composite obtained by polymer pyrolysis having 8%/volume porosity, and a Young's modulus of 50-70 GPa for a 2D CRFC-SiC composite processed by liquid silicon infiltration, having 5%/volume porosity and 40% fiber volume fraction. Nie *et al.* (2008) found a Young's modulus of 81 GPa for a 2D CFRC-SiC matrix, obtained by the CVI technique, having 40% volume fraction of carbon fibers and 11%/volume porosity. In the same work, stitched carbon fiber fabrics processed in the same way exhibited a Young's modulus of 62-64 GPa. Nie *et al.* (2009) also found a Young's modulus of 75±4 GPa for a needled carbon fiber densified with a SiC matrix, obtained by the CVI technique. These properties can be compared with those shown in Table 5 and Table 6. It is well known that Young's modulus of composites is mainly dependent on fiber's properties and on their volume fraction.

In the present work, the calculated elastic modulus for unidirectional CFRC-SiC composites, in the fiber direction, ranges from 140 to 170 GPa, and from 36 to 33 GPa, perpendicularly to fiber direction, considering a fiber volume fraction from 40 to 50%, respectively. For the 2D CFRC-SiC composite, elastic modulus range from 90 to 100 GPa, considering a fiber volume fraction from 40 to 50%. For the 2D CFRC-SiC composites having Z-reinforcement ranging from 2 to 6%, elastic modulus range from 80 to 95 GPa in the plane of reinforcement, and from 43 to 57 GPa in the out-of-plane direction, considering a total fiber volume fraction from 40 to 50% in the composite. Heidenreich (1997) resumed data on properties of C-SiC composites obtained from many companies. For instance, C-SiC composites obtained by CVI technique, having 45% fiber volume fraction, had the elastic modulus reported as 90-100 GPa. In the same work, C-SiC composites obtained by liquid polymer infiltration and by liquid silicon infiltration had the elastic modulus reported as 65 GPa (Heidenreich, 2007). Although these data are from various material types, it is possible to infer that the calculated values from this work, as shown in Tables 5 and 6, are reasonably in the range of those found in the literature.

The thermal properties depend on the axis of measurement and are mainly influenced by the carbon fiber. Calculation for thermal conductivity of CRFC-SiC composites (Table 6) are in the range from 32 to 46 W/m.K. The survey of properties of C-SiC composites, obtained by Heidenreich (1997), shows more conservative results for thermal conductivity, which are in the range of 5 to 20 W/m.K, although values up to 40 W/m.K can be found. These differences can be attributed to the differences on the manufacturing method and the presence of pores and microcracks which are not exactly accounted in Fabric Geometry Model. Thermal expansion coefficients calculated by the Fabric Geometry Model for

Table 6: Properties of 2.5D carbon fiber reinforced C/SiC hybrid matrix, having 5% volume porosity, from RT to 1200°C

Total fiber volume fraction (%)		**40**			**45**			**50**		
C/SiC matrix relation		**65/35**			**60/40**			**55/45**		
Fiber volume Z direction (%)		**2**	**4**	**6**	**2**	**4**	**6**	**2**	**4**	**6**
E_{11}		86	83	79	91	87	84	94	90	86
E_{22}		86	83	79	91	87	84	94	90	86
E_{33}		43	48	53	44	50	56	45	51	57
G_{12}		21	22	22	22	23	24	23	24	24
G_{23}		17	17	17	18	23	18	23	24	24
G_{13}		17	17	17	18	18	18	19	19	19
ν_{12}		0.09	0.11	0.13	0.09	0.11	0.12	0.08	0.10	0.12
ν_{13}		0.19	0.17	0.14	0.18	0.16	0.12	0.18	0.15	0.12
ν_{23}		0.19	0.17	0.14	0.18	0.16	0.14	0.18	0.15	0.13
k (W/mK)	k_{11}	46	45	45	42	42	42	40	39	39
	k_{22}	46	45	45	42	42	42	40	39	39
	k_{33}	37	37	39	34	35	36	32	33	34
α (10^{-6}/°C)	α_{11}	0.50	0.50	0.50	0.90	0.90	0.88	0.96	0.94	0.94
	α_{22}	0.50	0.50	0.50	0.90	0.90	0.88	0.96	0.94	0.94
	α_{33}	0.15	0.22	0.30	0.66	0.72	0.77	0.74	0.80	0.84

the CFRC-SiC composites are in the range of $0.15\text{x}10^{-6}$/°C to $1\text{x}10^{-6}$/°C, which are in the range ($-1\text{x}10^{-6}$/°C to $6\text{x}10^{-6}$/°C) showed in the survey of properties of C/SiC materials of the work of Heindenreich (2007).

CONCLUSION

This work described a simple method for estimation of the elastic and thermal properties of 2.5D carbon fiber reinforced carbon-SiC hybrid matrix composites. These materials are the state-of–the-art composites for use in thermal protection systems. The Z-direction reinforcement allows higher delamination resistance and endurance on thermal stresses generated by heat treatment processing, and also the interlaminar fracture toughness is improved. Mechanical and thermal properties of 2.5D CRFC-SiC composites were calculated based on composite average stiffness micromechanics. The modeling of properties by this simple method allows avoiding costly testing and reducing time consuming specimen preparation.

Mechanical properties of composites are fiber dominated. Calculations were done by considering the total carbon fiber volume fraction in the range of 40 to 50%, which are commonly found in carbon and ceramic composites reinforced with carbon fibers. The addition of only 2% of fibers out of the main plane of reinforcement increases the elastic modulus in the out-of-plane direction by about 20%.

An increase in the carbon fiber volume fraction from 40 to 50%, results in higher elastic properties, but nevertheless decreases the thermal conductivity. The calculated in-plane Young's modulus is in the range of 84 to 94 GPa, and the out-of-plane Young's modulus is in the range of 43 to 57 GPa. The calculated shear modulus is in the range of 17 to 24 GPa, regardless of the plane of shear stress.

The calculated thermal conductivity of 2.5D carbon fiber reinforced carbon-SiC hybrid matrix composites was found to be in the range of 32 to 46 W/m.K, and the calculated coefficient of thermal expansion was in between $0.15.10^{-6}$/°C to $0.94.10^{-6}$/°C. These results agreed in some extent to values found in the literature for similar materials.

The knowledge of the envelope of elastic and thermal properties of carbon and ceramic composites made with carbon fibers allows *ab initio* modeling thermoelastic properties of composites. These properties can be easily calculated by taking individual properties of each component (fiber and matrix) and the relative proportions of these components in the composite which is the essence of the homogenization theory.

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