






# Multi-objective Optimization Method For Repeat Ground-track Orbit Design Considering the Orbit Injection Error

Mingjun Pu<sup>1</sup>, Donghong Wang<sup>1</sup>, Yuanbo Wu<sup>1</sup>, Jihe Wang<sup>1</sup>, Xiaowei Shao<sup>1</sup>

Pu M  <https://orcid.org/0000-0002-7584-4342>  
Wang D  <https://orcid.org/0000-0001-7028-3120>  
Wu Y  <https://orcid.org/0000-0002-8155-4288>  
Wang J  <https://orcid.org/0000-0003-2792-4601>  
Shao X  <https://orcid.org/0000-0002-2603-057X>

## How to cite

Pu M; Wang D; Wu Y; Wang J; Shao X (2018) Multi-objective Optimization Method For Repeat Ground-track Orbit Design Considering the Orbit Injection Error. J Aerosp Technol Manag, 10: e2618. doi: 10.5028/jatm.v10.913.

**ABSTRACT:** Considering the adverse effects of orbit injection error, a novel repeat ground-track orbit redesign approach is proposed to reduce the fuel consumption caused by the orbital maneuvering from the injection orbit to the nominal orbit. By introducing the performance indexes of revisiting accuracy and orbit injection maneuvering fuel consumption, the problem of repeat ground track orbit redesign considering the orbit injection error is transformed into a multi-objective optimization problem, which can be solved by multi-objective genetic algorithm. Finally, the numerical simulations show that the redesigned repeat ground-track orbits not only can meet the requirements of revisiting ground targets with high accuracy, but also can reduce the required fuel consumption significantly.

**KEYWORDS:** Repeat ground-track orbit design, Orbit injection error, Revisiting accuracy, Fuel consumption, Multi-objective optimization.

## INTRODUCTION

Repeat ground-track (RGT) orbit is an orbit that retraces its ground track over a certain time interval, which can allow a satellite to repeatedly observe any particular spot within the predefined period of time, and has been employed in a number of Earth-observation and Earth-science missions, such as LANDSAT, SPOT, ENVISAT, RADARSAT and JASON (Nadoushan and Assadian 2015).

For RGT orbit design, the effect of non-spherical gravitational force cannot be neglected because this factor is the primary cause of the ground-track drift of low-Earth orbits (Fu *et al.* 2012). The gravitational second zonal harmonic  $J_2$  effect is usually used to reflect the main influence of the non-spherical gravitational force, based on the approaches for RGT orbit design, which can be categorized into two groups:

In the first group, the analytical theory is used to obtain RGT orbital elements. Tang and Liu (2013) consider the  $J_2$  effect and propose an orbit design method to find the RGT orbital elements within the range of a certain orbital altitude, which can achieve RGT orbit design based on given repeat cycle without changing the orbital inclination. Duan and Liu (2006) study the non-linear equations for RGT orbit design based on spherical geometry analysis, which can be combined with iteration method to obtain

1. Shanghai Jiao Tong University – School of Aeronautics and Astronautics – Aeronautical and Astronautical Science and Technology – Shanghai – China.

Correspondence author: Xiaowei Shao | Shanghai Jiao Tong University – School of Aeronautics and Astronautics – Aeronautical and Astronautical Science and Technology | Minhang District Dongchuan Road 800 | 200240 Shanghai – China | E-mail: shaowxmail@163.com

Received: Mar. 1, 2017 | Accepted: Jul. 10, 2017

Section Editor: Waldemar Leite Filho



the RGT orbital elements of semi-major axis and inclination. Aorpimai and Palmer (2007) consider  $J_2$  and  $J_4$  effect to propose a simple and readily implemented RGT orbit acquisition algorithm based on the nonsingular epicycle elements. Nadoushan and Assadian (2015) propose a novel method for RGT orbit design based on the Number Theory, where the concept of the sub-cycles is introduced to provide the optimal RGT orbits with desired revisit time and the optimal required tilt.

In the second group, the RGT design is described as a numerical optimization problem, and the numerical methods are used to obtain optimal RGT orbital elements. By analyzing the shortcomings of the Aorpimai and Palmer's research, Vtipil and Newman (2012) provide a quick and efficient orbit optimization design methodology to determine the accurate orbital characteristics of a RGT orbit for a given set of inclination, altitude of perigee, and repeat cycle. Saboori *et al.* (2013) study the relationships between repeat cycle and orbit revolutions of sun-synchronized RGT orbits, and choose objective functions among the orbital lifetime, revisit time and off-nadir resolution to obtain the optimal orbital elements. Abdelkhalik and Gad (2011) utilize RGT concept for designing natural orbits to visit target areas without the use of propulsion systems, in which the problem of optimal orbit design is described as an optimization problem and solved by single-objective genetic algorithm (SOGA).

Based on the literature review, it is concluded that previous RGT orbit design works are mainly based on the assumption that the satellite can be injected precisely, in which the analytic or numerical methods are used to obtain optimal orbital elements with a certain revisiting accuracy. However, due to the effects of rocket navigation accumulation error and propellant delivery unbalance existing in the process of satellite launch, there is always a certain orbit injection error between the injection orbits and the designed orbits (also called the nominal orbit) (Yang *et al.* 2016).

In the case of small orbit injection error, the satellite can be maneuvered from the injection orbits to the originally designed orbits to eliminate the adverse impacts of orbit injection error on orbit properties (Xu *et al.* 2016). However, with regard to the case of large orbit injection error, the aforementioned maneuvering strategy will bring in a lot of fuel consumption, which will impose an adverse impact on the orbital lifetime. Therefore, it is necessary to explore whether there exists some other RGT orbits in the vicinity of the injection orbit, of which the revisiting accuracy is similar to that of the originally designed orbit. Meanwhile, the redesigned RGT orbits are expected to consume less fuel required by orbit injection maneuver compared to the originally designed orbits.

Based on the aforementioned considerations, the problem of optimal RGT orbit design, taking into consideration the  $J_2$  effect and the orbit injection error, is addressed in this study. Besides, a kind of RGT orbit redesign approach is proposed to design nominal orbits that are expected to satisfy the requirements of revisiting ground targets accurately while reducing as much orbit injection maneuvering fuel consumption as possible, which can provide some references for the practical satellite launching mission deciders to choose the optimal nominal injection orbits.

The next section presents the problem statement, where the multi-objective optimization mathematical model for designing RGT orbit considering orbit injection error is developed. The objective functions of revisiting accuracy and orbit injection maneuvering fuel consumption are formulated in the section Optimization Objective Functions. In section Multi-Objective Genetic Algorithm for RGT orbit redesign, the developed solution algorithm for RGT redesign is presented, which is based on the multi-objective genetic algorithm (MOGA). In the section case studies, the redesigned RGT orbits, with repeat cycle of seven days, are presented to demonstrate the effectiveness of the proposed method. The last section concludes the paper.

---

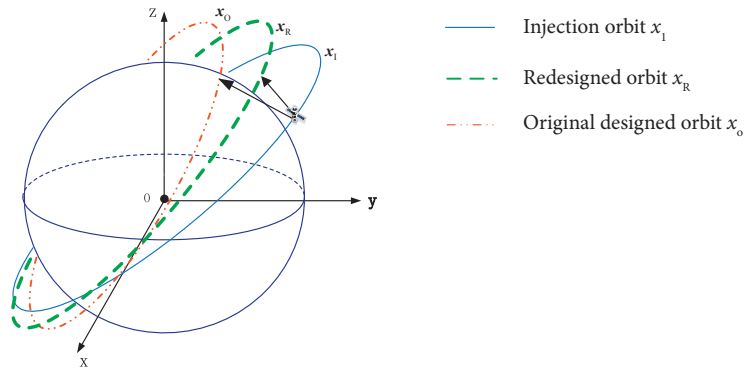
## PROBLEM STATEMENT

It is well known that there are many conflicting objectives that influence the selection of the nominal orbits (Saboori *et al.* 2013). As a result, there is an important research problem associated with satellite orbit design: how to design an optimal orbit to make all of these conflicting objectives as expected.

For the problem of redesigning RGT orbits to eliminate adverse impacts of orbit injection error, on the one hand, the revisiting accuracy of the redesigned RGT orbits should be no worse than that of the originally designed orbits, since it is the main criteria for judging the performance of RGT orbits. On the other hand, compared to the originally designed orbits, the redesigned RGT

orbits are expected to eliminate orbit injection error with less fuel expenditure. However, these two objectives may not be achieved simultaneously, because there are many RGT orbits that can meet the requirement of revisiting ground targets with high accuracy, while the fuel consumption required by the satellite maneuvering from the injection orbit to these orbits is difficult to determine.

Therefore, as shown in Fig. 1, the goal of this study is to find a new set of deterministic RGT orbital elements  $x_R$  in the vicinity of the injection orbit  $x_1$ , which will replace the originally designed RGT orbit  $x_o$  to be regarded as the actual nominal orbits of launched satellite, such that the fuel expenditure for maneuvering the satellite from the injection orbits to the nominal orbits is minimized.



**Figure 1.** Orbit maneuvering schematic diagram.

Based on the aforementioned considerations, the problem of redesigning RGT orbits can be described as a multi-objective optimization problem, of which the mathematical optimization model is formulated as (Eqs. 1 and 2):

$$\min_x G(\mathbf{x}) = \left\{ \lambda_1 \cdot \Delta r(\mathbf{x}), \lambda_2 \cdot \zeta(\mathbf{x} - \mathbf{x}_1) \right\} \quad (1)$$

subject to:

$$\mathbf{x} \in \mathbb{N}^{6 \times 1} : x_j^l \leq x_j \leq x_j^u \quad (j=1,2,3,4,5,6) \quad (2)$$

where  $\mathbf{x}$  is the design variable vector and each element  $x_j$  ( $j = 1, 2, 3, 4, 5, 6$ ) represents semi-major axis  $a$ , eccentricity  $e$ , inclination  $i$ , argument of perigee  $\omega$ , right ascension of the ascending node  $\Omega$ , mean anomaly  $M$  respectively;  $x_j^l$  and  $x_j^u$  are the lower and upper bounds of six components of design variables;  $\mathbf{x}_1$  is injection orbit;  $\Delta r(\mathbf{x})$  represents the objective function of revisiting accuracy;  $\zeta(\mathbf{x} - \mathbf{x}_1)$  represents the objective function of orbit injection maneuvering fuel consumption;  $\lambda_1$  and  $\lambda_2$  are the normalized weight coefficients, which can reflect the relative importance of two objectives and satisfy the relationships:  $0 \leq \lambda_1, \lambda_2 \leq 1$  and  $\lambda_1 + \lambda_2 = 1$ .

## OPTIMIZATION OBJECTIVE FUNCTIONS

The optimization objective functions for RGT orbit redesign are formulated in this section. Based on spherical geometry and orbital elements differential equations under  $J_2$  effect, the objective function of revisiting accuracy is presented at first. Then, the objective function of orbit injection maneuvering fuel consumption is established based on Gauss variation equation.

### OBJECTIVE FUNCTION 1: REVISITING ACCURACY

Revisiting accuracy is the main reference index to evaluate the properties of the designed RGT orbits. As shown in Fig. 2, the index of revisiting accuracy can be represented as the space distance  $\Delta r$  along the Earth surface between the sub-satellite point

A  $(\lambda_{T_0}, \varphi_{T_0})$  at the initial epoch  $T_0$  and the sub-satellite point A  $(\lambda_{T_f}, \varphi_{T_f})$  at the final epoch  $T_f$ ; and the smaller the value of  $\Delta r$ , the higher the revisiting accuracy of RGT orbit.

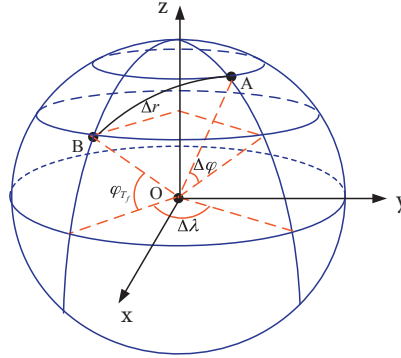


Figure 2. Orbit revisiting accuracy schematic diagram.

Based on the spherical geometry, the approximation calculation formula for  $\Delta r$  is given as:

$$\Delta r \approx \sqrt{(R_E \cdot \Delta \varphi)^2 + (R_E \cdot \cos \varphi_{T_f} \cdot \Delta \lambda)^2} \tag{3}$$

where  $R_E = 6378.136$  (km) is the equatorial radius;  $\Delta$  is the differential operator;  $\Delta \lambda$  and  $\Delta \varphi$  are the geocentric longitude and latitude difference between sub-satellite points A and B, respectively.

With Eq. 3, it is obvious that  $\Delta r$  is related to the location of the sub-satellite points A and B, which can be obtained based on the following calculation formulas:

Considering  $J_2$  effect, the differential equations of six orbital elements are (Eq. 4) (Cao *et al.* 2012):

$$\begin{cases} \frac{da}{dt} = \frac{de}{dt} = \frac{di}{dt} = 0 \\ \frac{d\Omega}{dt} = n_\Omega = -\frac{3n \cdot J_2 \cdot \cos i}{2(1-e^2)^2} \cdot \left(\frac{R_E}{a}\right)^2 \\ \frac{d\omega}{dt} = n_\omega = -\frac{3n \cdot J_2 \cdot (1-5\cos^2 i)}{4(1-e^2)^2} \cdot \left(\frac{R_E}{a}\right)^2 \\ \frac{dM}{dt} = n_M = n + \frac{3n \cdot J_2 \cdot (3\cos^2 i - 1)}{4(1-e^2)^{1.5}} \cdot \left(\frac{R_E}{a}\right)^2 \end{cases} \tag{4}$$

where  $n = \sqrt{\mu/a^3}$  is the mean angular velocity of satellite;  $\mu = 398600.4415$  (km<sup>3</sup> / s<sup>2</sup>) is the geocentric gravitational constant.

Given a set of initial orbital elements  $x = x_{T_0}$ , the orbital elements  $x_T$  corresponding to the end epoch of repeat cycle can be obtained (Eq. 5):

$$\begin{cases} a_T = a_{T_0}; e_T = e_{T_0}; i_T = i_{T_0} \\ \Omega_T = \Omega_0 + n_\Omega \cdot N_D \cdot P_E \\ \omega_T = \omega_0 + n_\omega \cdot N_D \cdot P_E \\ M_T = M_0 + n_M \cdot N_D \cdot P_E \end{cases} \tag{5}$$

where  $N_D$  is the repeat cycle;  $P_E = 86400$  (s) is the Earth rotation cycle. In addition, the relationship between true anomaly  $f$  and mean anomaly  $M$  is (Eq. 6) (Zhang 1998):

$$f \approx M + (2e - \frac{1}{4}e^3) \cdot \sin M + (\frac{5}{4}e^2 - \frac{11}{24}e^4) \cdot \sin 2M \quad (6)$$

Based on the obtained orbital elements, the corresponding position vector in the Earth Centred Inertial (ECI) frame can be calculated as follows (Eq. 7 and 8):

$$\mathbf{r}^{\text{ECI}} = \frac{a \cdot (1 - e^2)}{1 + e \cdot \cos \theta} \cdot \mathbf{M}^{\text{OI}} \cdot (\cos f, \sin f, 0)^{\text{T}} \quad (7)$$

$$\mathbf{M}^{\text{OI}} = \begin{bmatrix} C_{\omega} \cdot C_{\Omega} - C_i \cdot S_{\omega} \cdot S_{\Omega} & -S_{\omega} \cdot C_{\Omega} - C_i \cdot S_{\omega} \cdot C_{\Omega} & S_i \cdot S_{\Omega} \\ C_{\omega} \cdot S_{\Omega} + C_i \cdot S_{\omega} \cdot S_{\Omega} & -S_{\omega} \cdot S_{\Omega} + C_i \cdot C_{\Omega} \cdot C_{\omega} & -S_i \cdot C_{\Omega} \\ S_{\omega} \cdot S_i & C_{\omega} \cdot S_i & C_i \end{bmatrix} \quad (8)$$

where  $C = \cos$  and  $S = \sin$ .

To obtain the location of sub-satellite point,  $\mathbf{r}^{\text{ECI}}$  needs to be transformed into the corresponding position vector  $\mathbf{r}^{\text{ECEF}}$  in the Earth Centered Earth fixed (ECEF) frame, which is related to the theory of precession, nutation and rotation of Earth (IERS 2010). Finally, the longitude and latitude of the sub-satellite point can be obtained based on the spherical geometry (Eq. 9):

$$\lambda = \arctan\left(\frac{r_y^{\text{ECEF}}}{r_x^{\text{ECEF}}}\right); \quad \varphi = \arctan\left(\frac{r_z^{\text{ECEF}}}{\sqrt{(r_x^{\text{ECEF}})^2 + (r_y^{\text{ECEF}})^2}}\right) \quad (9)$$

## OBJECTIVE FUNCTION 2: ORBIT INJECTION MANEUVERING FUEL CONSUMPTION

To eliminate the adverse impacts of the orbit injection error, some velocity impulses are usually generated at the injection point to maneuver the satellite from the injection orbit to the originally designed orbit, which will bring in a certain fuel consumption (Rao 1978). In this section, the objective function of orbit injection maneuvering fuel consumption is established based on the amplitude of velocity pulses, because there is a mapping relationship between them (Wie and Barba 1985).

It is assumed that there is an orbit injection error  $\Delta \mathbf{x}$  between the injection orbital elements  $\mathbf{x}_1$  and the originally designed orbital elements  $\mathbf{x}_0$  (Eq. 10):

$$\Delta \mathbf{x} = \mathbf{x}_0 - \mathbf{x}_1 = (\Delta a, \Delta e, \Delta i, \Delta \omega, \Delta \Omega, \Delta M)^{\text{T}} \quad (10)$$

The eccentricity error vector and inclination error vector are respectively represented as  $\Delta \mathbf{e}$  and  $\Delta \mathbf{i}$  (Eqs. 11 and 12) (D'Amico 2005):

$$\Delta \mathbf{e} = \begin{Bmatrix} \Delta e_x \\ \Delta e_y \end{Bmatrix} = \begin{Bmatrix} e_o \cdot \cos \omega_o - e_l \cdot \cos \omega_l \\ e_o \cdot \sin \omega_o - e_l \cdot \sin \omega_l \end{Bmatrix} \quad (11)$$

$$\Delta \mathbf{i} = \begin{Bmatrix} \Delta i_x \\ \Delta i_y \end{Bmatrix} \approx \begin{Bmatrix} i_o - i_l \\ (\Omega_o - \Omega_l) \cdot \sin i_l \end{Bmatrix} \quad (12)$$

Based on the works of D'Amico and Montenbruck (2006), the simplified Gauss variation equations applicable for near-circular orbits are presented as follows:

$$\begin{Bmatrix} \Delta a \\ \Delta e_x \\ \Delta e_y \\ \Delta i_x \\ \Delta i_y \\ \Delta u \end{Bmatrix} = \frac{1}{v} \cdot \begin{bmatrix} 0 & 2a & 0 \\ \sin u & 2 \cos u & 0 \\ -\cos u & 2 \sin u & 0 \\ 0 & 0 & \cos u \\ 0 & 0 & \sin u \\ -2 & \frac{-3v \cdot \Delta t}{a} & -\sin u \cot i \end{bmatrix} \cdot \begin{Bmatrix} \Delta v_R \\ \Delta v_T \\ \Delta v_N \end{Bmatrix} \quad (13)$$

where  $v$  is satellite velocity;  $\Delta u = \Delta \omega + \Delta M$  is argument of latitude error between  $x_1$  and  $x_0$ ;  $\Delta v_R$ ,  $\Delta v_T$  and  $\Delta v_N$  are the radial, in-track and cross-track velocity pulse, respectively;  $\Delta t$  is the time interval between adjacent in-track velocity pulses.

According to Eq. 13, the required velocity pulses of maneuvering the satellite from the injection orbit  $x_1$  to the designed orbit  $x_0$  can be obtained, and the amount of fuel consumption can be represented as the sum of the velocity pulses amplitudes.

### In-Plane Velocity Pulses Correction

With Eq. 13, it is obvious that the in-plane errors  $\Delta a$ ,  $\Delta e$  and  $\Delta u$  can be corrected by the radial pulse  $\Delta v_R$  and the in-track pulse  $\Delta v_T$ . The relationship between the in-plane pulses and orbital elements error is derived as follows (Eq 14):

$$\begin{cases} \Delta a = \frac{2a}{v} \cdot \Delta v_T, \\ \Delta e_x = \frac{2 \cos u}{v} \cdot \Delta v_T + \frac{\sin u}{v} \cdot \Delta v_R, \\ \Delta e_y = \frac{2 \sin u}{v} \cdot \Delta v_T - \frac{\sin u}{v} \cdot \Delta v_R, \\ \Delta u = -\frac{3 \Delta t}{a} \cdot \Delta v_T - \frac{2}{v} \cdot \Delta v_R - \frac{\sin u \cdot \cot i}{v} \cdot \Delta v_N. \end{cases} \quad (14)$$

Since the correction efficiency of  $\Delta v_R$  is only half of that of  $\Delta v_T$ , the former is usually not considered to correct the in-plane error. In this study, the three in-track pulses method is used to solve the Eq. 14, which is derived by Zeng *et al.* (2012). The formulas for calculating three in-track correction velocity pulses  $\Delta v_{T1}$ ,  $\Delta v_{T2}$  and  $\Delta v_{T3}$  are given as:

$$\begin{cases} \Delta v_{T1} = -\frac{v}{8a} (\Delta a - a \cdot \delta e) - \frac{a \cdot \Delta u}{3P}, \\ \Delta v_{T2} = \frac{v}{4a} (\Delta a - a \cdot \delta e), \\ \Delta v_{T3} = -\frac{v}{8a} (\Delta a - a \cdot \delta e) + \frac{v}{2a} \Delta a + \frac{a \cdot \Delta u}{3P}. \end{cases} \quad (15)$$

where  $\delta e$  is the module of the eccentricity error vector:  $\delta e = \|\Delta e\| = \sqrt{\Delta e_x^2 + \Delta e_y^2}$ ;  $P$  is the nodal period, representing the time interval between two passages at the ascending node or the descending node:  $P = N_D / N_O$  ( $N_O$  is the orbit revolutions).

According to Eq. 15, the estimated fuel consumption  $\zeta_T$  for correcting the in-plane error can be obtained as follows (Eq. 16):

$$\zeta_T = \|\Delta v_{T1}\| + \|\Delta v_{T2}\| + \|\Delta v_{T3}\|. \quad (16)$$

### Out-Of-Plane Velocity Pulses Correction

With Eq. 13, the out-of-plane error  $\Delta i$  can be corrected by the cross-track pulse  $\Delta v_N$ :

$$\Delta v_N = v \cdot \delta i \quad (17)$$

where  $\delta i$  is the module of inclination error vector, i.e.,  $\delta i = \sqrt{\Delta i^2 + (\Delta \Omega \cdot \sin i)^2}$ .

According to Eq. 17, the estimated fuel consumption  $\zeta_N$  for correcting the out-of-plane error is (Eq. 18):

$$\zeta_N = \|\Delta v_N\| \quad (18)$$

Based on the aforementioned formulas, the total estimated fuel consumption  $\zeta$  for orbit injection maneuvering can be calculated as (Eq. 19):

$$\zeta = \zeta_T + \zeta_N \quad (19)$$

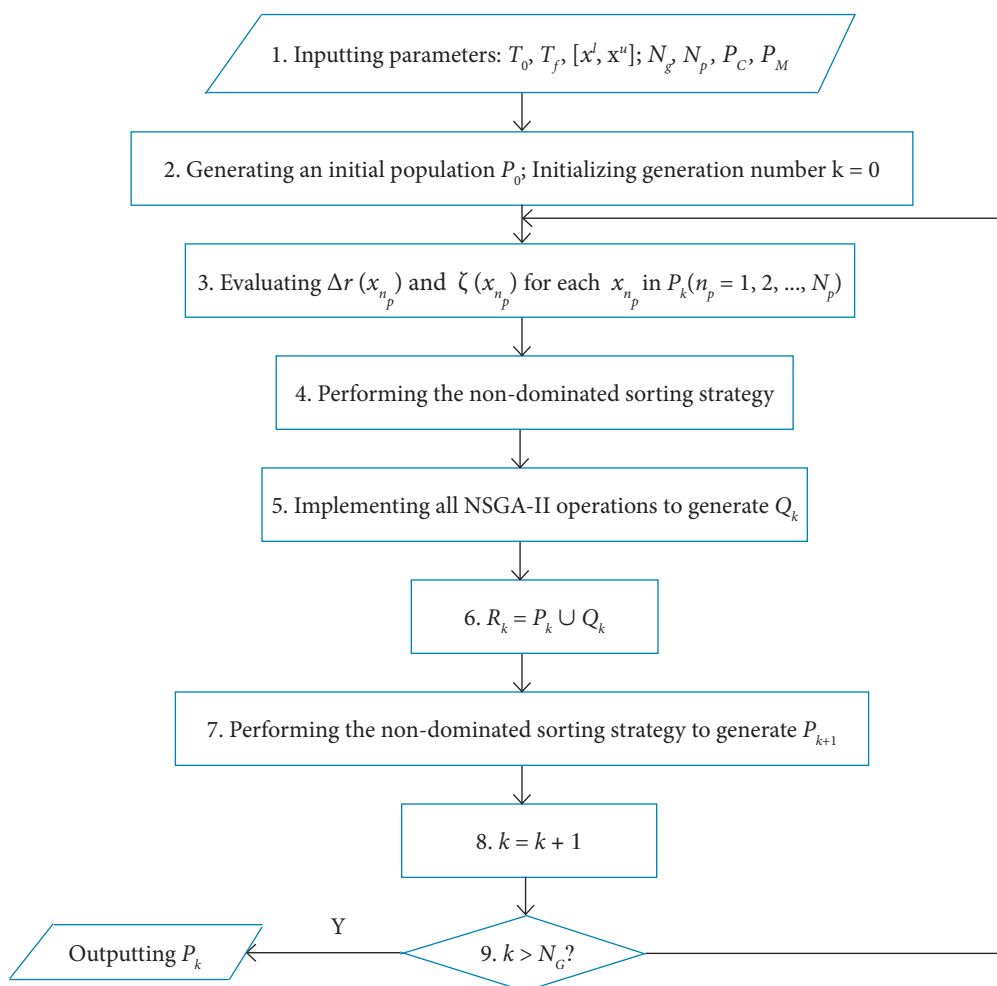
## MULTI-OBJECTIVE GENETIC ALGORITHM FOR RGT ORBIT REDESIGN

Based on the developed multi-objective optimization model, the problem of RGT orbit redesign is transformed into a multi-objective optimization problem indeed. As a result, the multi-objective stochastic optimization algorithm to solve this problem is introduced in this section, and the fuel-optimal design for RGT orbits is further explored based on this stochastic optimization algorithm.

Multi-objective evolutionary algorithm (MOEA) is a kind of stochastic global optimization method by simulating natural evolutionary process, which is widely used to solve multi-objective optimization problems (MOP) because of its inherent parallelism and randomness (Tan *et al.* 2012). Compared to other MOEA, such as multi-objective genetic algorithm (MOGA), niched Pareto genetic algorithm (NPGA-II), and strength Pareto evolutionary algorithm (SPEA-II), the Elitist Non-dominated Sorting Genetic Algorithm (NSGA-II) has revealed a better performance in convergence and uniform distribution, which uses not only an elite-preserving strategy but also an explicit diversity-preserving mechanism (Li and Zhang 2009). In NSGA-II, a solution is evaluated as good or bad based on its non-dominance ranking  $NDR$  and local crowding distance  $LCD$  in the population, and an optimal Pareto set containing  $N_p$  non-dominated solutions (also called Pareto optimal solutions) can be obtained based on this algorithm (Deb *et al.* 2002).

Based on the principle of the NSGA-II, the procedures of the fuel-optimal RGT orbit redesign method are presented as follows:

1. Inputting the simulation parameters including the initial epoch  $T_0$  and final epoch  $T_f$  of orbit design, the optimization range of design variables  $[x^l, x^u]$ , genetic parameters (including the maximum optimization generations  $N_G$ , population size  $N_p$ , crossover probability  $P_c$  and mutation probability  $P_m$ );
2. Initializing generation number  $k = 0$ , and generating an initial population  $P_0$  containing  $N_p$  individuals  $x_{n_p} \in [x^l, x^u]$  ( $n_p = 1, 2, \dots, N_p$ );
3. Evaluating  $\Delta r(x_{n_p})$  and  $\zeta(x_{n_p})$  for each  $x_{n_p}$  in  $P_k$  based on Eqs. 3 and 19;
4. Performing the non-dominated sorting strategy to sort  $P_k$  based on  $\Delta r$  and  $\zeta$ ; and calculating  $NDR$  and  $LCD$  for each individual in  $P_k$ ;
5. Implementing all NSGA-II operations including tournament selection, genetic operation (including simulated binary (SBX) crossover and polynomial mutation), to generate an offspring population  $Q_k$ ;
6. Recombining the  $P_k$  and  $Q_k$  to generate the population  $R_k = P_k \cup Q_k$ ;
7. Performing the non-dominated sorting strategy to  $R_k$  to obtain a new population  $P_{k+1}$ , which is filled with Pareto feasible solutions;
8. Updating the generation number  $k = k + 1$ ;
9. Applying stopping criterion: if the maximum generation number  $N_G$  is not reached ( $k \leq N_G$ ), repeat steps 3 to 8 until the termination criterion is met. The flow chart of the fuel-optimal RGT redesign is presented in Fig. 3.



**Figure 3.** Flow chart of RGT orbit redesign considering injection error.

## CASE STUDIES

In this section, the example of RGT orbit design with a given repeat cycle is presented to demonstrate that the proposed RGT orbit redesign method can effectively reduce the orbit injection maneuvering fuel consumption. The procedures of simulation are presented as follows:

First, based on the ideal RGT orbit design method proposed by D'Amico *et al.* (2004), a set of RGT orbital elements  $x_0$  is presented, and it is regarded as the originally nominal orbit for the launched satellite. Then, the injection orbital elements  $x_1$ , based on the given orbit injection error, is calculated. According to the formulated objective functions, the revisiting accuracy of  $x_0$  and  $x_1$  and the fuel consumption caused by maneuvering the satellite from  $x_1$  to  $x_0$  are analyzed respectively. Finally, the redesigned RGT orbital elements  $x_R$  considering different objective weight coefficients, which will be used as the actual nominal orbit for the launched satellite, are presented. By comparing the revisiting accuracy, as well as the orbit injection maneuvering fuel consumption corresponding to  $x_0$  and  $x_R$ , the effectiveness of the proposed method for RGT orbit redesign considering orbit injection error is demonstrated.

Considering  $J_2$  effect, the simulation parameters used in this study are listed in Table 1, and the computational time of per generation is  $\sim 30$  s on 3.2 GHz Xeon processor.



**Table 1.** Fundamental simulation parameters.

Parameters	Symbol	Value	Unit
Repeat cycle	$N_D$	7	(days)
Orbit revolutions	$N_O$	111	(orbits)
Initial epoch	$T_0$	Jan 1, 2018 12:00:00.000 UTCG	
Final epoch	$T_f$	Jan 8, 2018 12:00:00.000 UTCG	
Maximum genetic generation	$N_G$	300	--
Population size	$N_p$	25	--
Crossover probability	$P_c$	0.9	--
Mutation probability	$P_m$	0.1	--

### IDEAL RGT ORBIT DESIGNED WITHOUT CONSIDERING THE ORBIT INJECTION ERROR

According to the predefined simulation parameters and the conventional method for RGT orbit design, a set of ideal RGT orbital elements  $x_o$  without considering the orbit injection error is presented. Then, as referred to the orbit injection error data provided by the works of D'Amico (2005), the injection error of six orbital elements is given as follows:  $\Delta a = 2$  (km),  $\Delta e = 0.0015$ ,  $\Delta i = 0.05$  (°),  $\Delta \omega = 1$  (°),  $\Delta \Omega = 0.05$  (°),  $\Delta M = 1$  (°), based on which the injection orbital elements  $x_1$  can be calculated.

Table 2 lists the obtained orbital elements of  $x_o$  and  $x_1$ . Based on the formulation of objective functions, the revisiting accuracy of these two kinds of orbits and the fuel consumption caused by maneuvering the satellite from  $x_1$  to  $x_o$  are analyzed respectively and listed in Table 2.

From Table 2, it is known that under the influence of given orbit injection error, the revisiting accuracy of the nominal orbit degrades from 0.127 (m) to 1.987 (km), which cannot meet the requirements of revisiting ground targets with high accuracy. If the maneuvering strategy of maneuvering the satellite from the injection orbit  $x_1$  to the originally designed orbit  $x_o$  is adopted, the corresponding orbit injection maneuvering fuel consumption reaches 42.687 (m / s), which would seriously affect the orbital lifetime and functional characteristics of the launched satellite with very limited amount of fuel.

**Table 2.** Originally designed and injection RGT orbit.

Parameters	$x_o$	$x_1$	Unit
$a$	6686.219636	6688.219636	(km)
$e$	0.00110943034	0.00260943034	--
$i$	96.70533905	96.75533905	(°)
$\omega$	90	91	(°)
$\Omega$	11.25649204	11.30649204	(°)
$M$	270	271	(°)
$\Delta r$	<b>0.127</b>	<b>1.987×10<sup>3</sup></b>	(m)
$\zeta$	<b>22.687</b>	--	(m·s <sup>-1</sup> )

As a result, it is quite necessary to explore whether there exists some other RGT orbits  $x_r$  with similar revisiting accuracy to that of the originally designed orbit  $x_o$ , which can reduce the orbit injection maneuvering fuel consumption drastically to be regarded as the actual nominal orbit for the launched satellite.

### OPTIMAL RGT ORBIT DESIGN CONSIDERING THE ORBIT INJECTION ERROR

In this section, the proposed RGT orbit redesign approach is used to generate fuel-optimal RGT orbits based on three different sets of objective weight coefficients, which are expected to achieve the goal of revisiting ground targets with high accuracy while

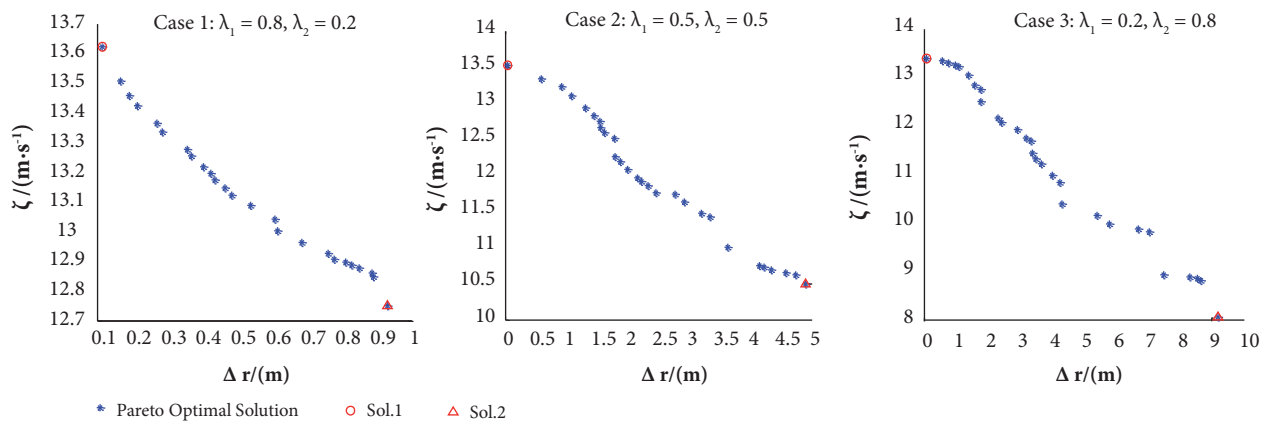
reducing as much orbit injection maneuvering fuel consumption as possible. Given that the redesigned orbits should be in the vicinity of the injection orbit to reduce utmost maneuvering fuel consumption, the optimization range of six design variables is given in Table 3. Based on the MOGA's parameters configuration and different objective weight coefficients setting, the Pareto fronts including all the orbits achieving the goal of optimal RGT orbit design are presented in Fig. 4. The most remarkable solutions of the obtained Pareto fronts are the solutions at its both ends, which are marked as *Sol. 1* and *Sol. 2*, respectively.

**Table 3.** Optimization range of design variables.

Parameters	Upper bound	Lower bound	Unit
$a$	6690	6686	(km)
$e$	0.004	0	--
$i$	97	96.5	(°)
$\omega$	93	89	(°)
$\Omega$	11.5	11	(°)
$M$	273	269	(°)

Table 4 lists the orbital elements, revisiting accuracy and the orbit injection maneuvering fuel consumption corresponding to corner solutions *Sol. 1* and *Sol. 2* obtained in three different cases. There are some analyses for Fig. 4 and Table 4, as follows:

1. For three different sets of objective weight coefficients, all of optimal RGT orbits designed from the proposed multi-objective optimization design approach can achieve the goal of revisiting ground sites precisely. Meanwhile, the required maneuvering fuel consumption can be significantly reduced;
2. Compared to the originally designed orbit  $x_o$  provided by the conventional RGT orbit design approach, the optimal orbit solutions *Sol. 1* provided by the proposed RGT orbit redesign method can achieve better revisiting accuracy, while the required orbit injection maneuvering fuel consumption can be reduced by approximately 40%. Based on this analysis, the effectiveness of the proposed RGT orbit redesign method is demonstrated;
3. With the increase of objective weight  $\lambda_2$ , the required orbit injection maneuvering fuel consumption corresponding to the redesigned RGT orbits is reduced. Compared to the originally designed orbit, the optimal orbit solutions *Sol. 2* under the case of  $\lambda_2 = 0.2, 0.5, 0.8$  can reduce the maneuvering fuel consumption by 43.7%, 53.7% and 64.4%, respectively. Therefore, it can be inferred that with the further increase of  $\lambda_2$ , the fuel consumption to eliminate the adverse impacts of orbit injection error will be further decreased.



**Figure 4.** Pareto front of optimal RGT orbits based on three groups of objective function weight coefficients.

**Table 4.** Corner solutions of Pareto fronts in three different cases.

Parameters	Case 1		Case 2		Case 3		Unit
	Sol. 1	Sol. 2	Sol. 1	Sol. 2	Sol. 1	Sol. 2	
$a$	6686.222532	6686.222493	6686.222529	6686.222515	6686.222523	6686.222456	(km)
$e$	0.0028320334	0.0028125730	0.0028292184	0.0028104857	0.0028318135	0.0027699246	--
$i$	96.70496866	96.70496549	96.70497046	96.70497075	96.70497123	96.70497759	(°)
$\omega$	89.75036429	89.84617198	89.74041252	89.91296580	89.76716640	96.70497759	(°)
$\Omega$	11.31475255	11.31804333	11.31492998	11.31527081	11.32086173	11.31672577	(°)
$M$	271.9293670	271.8980275	271.9320545	271.9835624	271.8988329	271.9988570	(°)
$\Delta r$	<b>0.0191</b>	<b>0.931</b>	<b>0.0342</b>	<b>4.894</b>	<b>0.0733</b>	<b>9.198</b>	(m)
$\zeta$	<b>13.624</b>	<b>12.749</b>	<b>13.526</b>	<b>10.495</b>	<b>13.397</b>	<b>8.075</b>	(m·s <sup>-1</sup> )

## CONCLUSIONS

In this study, the problem of redesigning RGT orbits to overcome the adverse effects of orbit injection error is addressed. To find the optimal RGT orbits that can achieve the goal of revisiting ground targets with high accuracy while reducing as much orbit injection maneuvering fuel consumption as possible, a multi-objective optimization method for RGT orbit design is developed. The numerical results are provided to demonstrate that the RGT orbits designed by the proposed multi-objective optimization method outperform the ones designed from the traditional RGT orbit design method without considering the orbit injection. In addition, the influence of different objective weights on designed orbit solutions has been explored, which can be helpful for designers to make a proper trade-off between two objectives to meet different mission requirements.

## AUTHOR'S CONTRIBUTION

Conceptualization, Pu M and Wang J; Methodology, Pu M; Investigation, Pu M, Wang D and Wu Y; Writing – Original Draft, Pu M; Writing – Review and Editing, Pu M, Wang D and Wu Y; Funding Acquisition, Shao X and Wang J; Resources, Shao X and Wang J; Supervision, Shao X.

## REFERENCES

- Abdelkhalik O, Gad A (2011) Optimization of space orbits design for Earth orbiting missions. *Acta Astronautica* 68(7-8):1307-1317. doi: 10.1016/j.actaastro.2010.09.029
- Aorpimai M, Palmer PL (2007) Repeat-groundtrack orbit acquisition and maintenance for Earth-observation satellites. *Journal of guidance, control, and dynamics* 30(3):654-659. doi: 10.2514/1.23413
- Cao HW, Liu Z (2012) Accuracy assessment of satellite orbit prediction based on J2 perturbation model. *Remote Sensing Information* 6:21-27. doi: 10.3969/j.issn.1000-3177.2012.06.004
- D'Amico S (2005) Flight dynamics operations for TanDEM-X formation flying. DLR-GSOC TN, 05-03. Deutsches Zentrum für Luft und Raumfahrt, Oberpfaffenhofen.
- D'Amico S, Arbinger C, Kirschner M, Campagnola S (2004) Generation of an optimum target trajectory for the TerraSAR-X repeat observation satellite. Presented at: 18th International Symposium on Space Flight Dynamics (ESA SP-548). Proceedings of 18th International Symposium on Space Flight Dynamics; Munich, Germany 548:137.

- D'Amico S, Montenbruck O (2006) Proximity operations of formation-flying spacecraft using an eccentricity/inclination vector separation. *Journal of Guidance, Control, and Dynamics* 29(3):554-563. doi: 10.2514/1.15114
- Deb K, Pratap A, Agarwal S, Meyarivan T (2002) A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE transactions on evolutionary computation* 6(2):182-197. doi: 10.1109/4235.996017
- Duan F, Liu J (2006) Calculation method of a kind of near-round repeating satellite orbit using nadir points. *Chinese Space Science and Technology* 26(3):38-43. doi: http://10.3321/j.issn:1000-758X.2006.03.007
- Fu X, Wu M, Tang Y (2012) Design and maintenance of low-Earth repeat-ground-track successive-coverage orbits. *Journal of Guidance, Control, and Dynamics* 35(2):686-691. doi: 10.2514/1.54780
- Li H, Zhang Q (2009) Multiobjective optimization problems with complicated Pareto sets, MOEA/D and NSGA-II. *IEEE Transactions on Evolutionary Computation* 13(2):284-302. doi: 10.1109/TEVC.2008.925798
- Nadoushan MJ, Assadian N (2015) Repeat ground track orbit design with desired revisit time and optimal tilt. *Aerospace Science and Technology* 40:200-208. doi: 10.1016/j.ast.2014.11.007
- IERS conventions (2010) IERS Technical note n° 36. Petit G, Luzum B, editors. Frankfurt am Main: Verlag des Bundesamts für Kartographie und Geodäsie.
- Rao PP (1978) Monte Carlo analysis of impulse requirements for injection error correction. *Journal of Guidance, Control, and Dynamics* 1(4):225-231.
- Saboori B, Bidgoli AM, Saboori B (2013) Multiobjective optimization in repeating sun-synchronous orbits design for remote-sensing satellites. *Journal of Aerospace Engineering* 27(5):04014027. doi: 10.1061/(ASCE)AS.1943-5525.0000340
- Tan YY, Jiao YC, Li H, Wang XK (2012) MOEA/D-SQA: a multi-objective memetic algorithm based on decomposition. *Engineering Optimization* 44(9):1095-1115. doi: 10.1080/0305215X.2011.632008
- Tang M, Liu Y (2013) The design of recursive orbit during deorbit. *Aerospace Control* 6:62-70. doi: http://10.3969/j.issn.1006-3242.2013.06.012
- Vtipil S, Newman B (2012) Determining an Earth observation repeat ground track orbit for an optimization methodology. *Journal of Spacecraft and Rockets* 49(1):157-164. doi: 10.2514/1.A32038
- Wie B, Barba PM (1985) Quaternion feedback for spacecraft large angle maneuvers. *Journal of Guidance, Control, and Dynamics* 8(3):360-365. doi: 10.2514/3.19988
- Xu XJ, Lin RC, Ma LH, Ai GX (2016) The Orbit Evolution of the iHCO Satellite in the CAPS System. *Astronomical Research and Technology* 1:70-74. doi: 10.3969/j.issn.1672-7673.2016.01.008
- Yang F, Zhang XD, Liu C, Mao CH, Hu Z (2016) Effect study of equalizing expulsion on precision of satellite trajectories based on upper stage parallel tank [J]. *Aerospace Shanghai* 1:60-65 doi: 10.19328/j.cnki.1006-1630.2016.S1.011
- Zeng G, Hu M, Yao H (2012) Relative orbit estimation and formation keeping control of satellite formations in low Earth orbits. *Acta Astronautica* 76:164-175. doi: 10.1016/j.actaastro.2012.02.024
- Zhang RW (1998) *Satellite orbit attitude dynamics and control* [M]. Beijing: University of Aeronautics and Astronautics Press.